THE APPLICATION OF ADJUSTMENT COMPUTATIONS IN COASTAL HYDROGRAPHIC SURVEYING

by

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1. INTRODUCTION

1.1 PURPOSE.

The purpose of this investigation is to show how adjustment computations may be applied to certain coastal hydrographic surveying operations. Particular emphasis is placed upon the determination of shore positions using information obtained aboard the survey vessel from visual and electronic positioning systems. The inverse problem, determination of the position of the ship by means of observations made at known shore positions, is also treated with an adjustment computation procedure.

The presently employed visual and electronic methods of location in coastal hydrographic surveying are described in order to illustrate the particular problems to which the adjustment procedures are to be applied. The instruments and equipment currently utilized to obtain positioning information are explained to indicate their relative merits and the accuracies that can be expected with them.

A brief summary of the state of automation in hydrographic surveying is presented to illustrate the feasibility of incorporating automated adjustment procedures into these operations on a real-time basis.

After this background information has been presented, the adjustment procedures are described for the two general problems. In each instance the overall adjustment procedure to be employed is outlined using standard matrix notations. The use of matrix algebra was decided

upon because of its straight forward adaptation to high-speed digital computer operations. Next, the various equations and relationships pertaining to the particular problem being discussed are developed in detail. The error analysis method for each problem is then explained. Finally, a numerical example of each adjustment is presented to illustrate the application of the procedure to a hydrographic survey operation. These numerical examples are solved by computer programs which are designed to determine all necessary auxiliary information and perform the required adjustment and error analysis computations. Particular attention has been given to developing the programs in as general a form as possible to allow for a wide latitude in the type and amount of observational input information.

Since real input data could not be obtained, simulated information was compiled for each problem using plotting sheets and equipment provided by the U.S. Naval Oceanographic Office. For this reason the accuracies of determinations must not be construed as being representative of results which would be obtained with the use of actual observational data. The primary objective of the numerical examples is to illustrate the methods in which the adjustment computations would be applied to coastal survey problems.

1.2 APPLICATIONS.

1.2.1 Shipboard Observation Application.

A procedure is developed which permits the determination of ship and shore positions using only shipboard information. The data provided by the solution of this problem are utilized for charts to aid

in the conduct of navigation and offensive military operations along inaccessible coastal regions. This application is designed in the form of a rapid self-contained positioning system which will enable the survey vessel to locate itself with relation to the coastal topography without placing personnel or equipment ashore. It could find particular use as a method for conducting surveys in hostile environments. As such, this procedure represents a possible method for arriving at a solution to certain special coastal hydrographic survey problems described by THOMAS(1).

The problem is dealt with in a completely generalized form in this investigation. That is, it is assumed that no topographic survey data is available concerning the positions of terrestrial objects. Further, it is assumed that the coast is inaccessible, either by virtue of impenetrable natural obstacles or, more likely, because the territory is occupied by an unfriendly nation. A second objective of the hydrographic survey then, in addition to obtaining the usual ocean depth information, is to produce data for use in developing a rough topographical survey; to the extent of determining the positions of prominent natural and man-made objects. These terrestrial points would later be used by combatant ships for coastal navigation and as references for offshore positioning preparatory to offensive bombardment operations against coastal targets.

As a modification to the completely generalized solution, the situation frequently arises wherein limited positional information is available concerning terrestrial objects. This information, combined with the shipboard observations, could be utilized to determine the

locations of newly constructed buildings such as enemy fortifications, as well as the usual ship positions, with considerably more accuracy than would be possible with the completely generalized problem.

1.2.2 Shore-based Observation Application.

The primary application of the ship positioning information provided by visual shore observations is for the precise location of soundings. These data are subsequently utilized for the compilation of accurate depth information on nautical charts. Other uses of this method of accurate ship positioning include geophysical surveys, planning for offshore military defense and civil engineering projects, location of submerged objects, cable laying operations, harbor improvement planning and oceanographic studies.

Since this method yields redundant information, an adjustment is performed to obtain the best possible ship position. The results obtained for ship positions along a considerable length of the coast, together with the positions as determined by the ship itself simultaneously from an electronic system, can provide the information required to calibrate and check the accuracy of the electronic system. Such comparisons could disclose systematic errors in electronic lattices along the coast which would reduce their accuracy. OSBORN(2) discusses in detail this method of determining the accuracy of and calibrating an electronic surveying system at the time of initial installation.

2. HYDROGRAPHIC SURVEYING

2.1 GENERAL.

Hydrographic surveying consists of the measurement of ocean depths and determination of the nature of the sea floor. The locations of certain features of adjoining coastal areas are also ascertained for use in aiding coastal navigation. The hydrographic survey can include gravimetric, tidal, current and meteorological data measurements as well as the usual position and sounding information.

The positions at which depth measurements are made are usually determined by reference to established points on shore. This procedure enables the offshore points to be located at their correct geographic positions, and insures that the land and marine features are in proper relationship to each other. The principal object of obtaining water depth and bottom composition and configuration information by hydrographic surveying is to enable the compilation of nautical charts and related publications for the mariner(3).

Depth or sounding information is obtained almost exclusively with the electronic echo sounder. Only in extremely shallow and inaccessible water areas are lead lines and sounding poles used in place of echo sounding devices. Echo sounding equipment of the recording type is usually used since it provides a graphic profile of the ocean floor, enabling the detailed charting of submarine relief.

2.2 COASTAL HYDROGRAPHY.

This investigation is concerned with hydrographic survey opera-

tions which are accomplished along the coast in sight of land. This branch of hydrographic surveying involves the determination of the shoreline, limited coastal topographic features, and the ocean bottom configuration in the immediate vicinity of the coast. The marine region encompasses areas extending to ranges of ten to fifteen miles from the shoreline.

Normally, prior to the conduct of the coastal hydrographic survey, a geodetic control survey is accomplished in order to establish control for shore positions. These shore points are used either to position target signals for visual sighting from the survey ship or for the location of sites for electronic positioning system transmitters. The usual interval between signals for coastal surveys is one to two miles. The distances may be reduced to one-half to one mile for large scale harbor surveys.

Small launches, known as sounding boats, are utilized to obtain depth measurements close to shore. In areas where the bottom slopes gradually, systems of sounding lines are normally taken parallel to the coast. The spacing between lines is least nearest the shoreline, usually less than fifty yards. It is increased outward from the shore. The spacing depends primarily on the depth of water and the type of bottom.

In addition to the survey of coastal waters, information is obtained on the shoreline itself and immediately adjacent land areas and outlying islands. This coastal mapping is generally limited to terrain features needed for control of the survey. Conspicuous landmarks which can be used by the mariner as aids in alongshore navigation are also

accurately located for inclusion on the nautical chart(4).

The most economical method for obtaining the preliminary shoreline and coastal topographic data, when it is feasible, is by means of aerial photography. Large-scale photogrammetric shoreline manuscript maps are produced to furnish advance reconnaissance information and provide detail to aid the hydrographer in mapping the coastline and terrestrial features.

2.3 VISUAL POSITIONING.

Various procedures are utilized to accurately determine the position of the survey craft from which depth and other measurements are made. The two general methods of hydrographic control most commonly used are visual and electronic.

Visual control is normally the method employed when surveying is performed within sight of the coast. Visual reference is made to conspicuous landmarks; objects such as geodetic survey control signals, prominent natural or man-made objects and supplementary stations established especially for the hydrographic survey(see Figure 2.1). The supplementary signals are positioned in relation to geodetic control stations by intersection, resection or traverse methods.

The positioning of a survey vessel by visual means usually falls within one of the three following categories:

(1) Resection from the ship.

(2) Intersection from the shore.

(3) Simultaneous intersection-resection.

A number of procedures are available to the hydrographic surveyor by

Figure 2.1. Hydrographic signal erected at a known shore point for use in controlling a coastal survey.

which he may apply the three basic methods. The requirements of the particular survey, primarily the resources available for its conduct and the accuracy specified for the results, dictate the particular procedure to be employed. The environment of the area to be surveyed also influences the specific methods to be utilized in accomplishing the project.

Both of the problems presented in this investigation depend upon visual observations for an integral part of their input information. Also, both utilize certain of the basic concepts of visual location methods for their solution. For these reasons and in order to provide suitable background information for the investigation, certain visual location and solution methods are described below.

2.3.1 Location Methods.

(a) By two angles from the ship.

This is the most common method used in conventional coastal surveying operations. Visual reference is made to conspicuous landmarks of known location along the shore. Usually two horizontal sextant angles, designated α and β in Figure 2.2, are observed simultaneously from the ship in the manner indicated in Figure 2.3. If conditions permit, a third "check angle", the angle Θ subtended by the extreme points T and V in Figure 2.2, is also observed to assist in detecting gross errors. A less accurate observation method entails the use of compass bearings to the shore objects instead of sextant angle cuts. In practice, a mechanical solution for the resultant ship's position is obtained with a device known as the three-arm protractor. This procedure as well as the graphical and analytical solutions are described

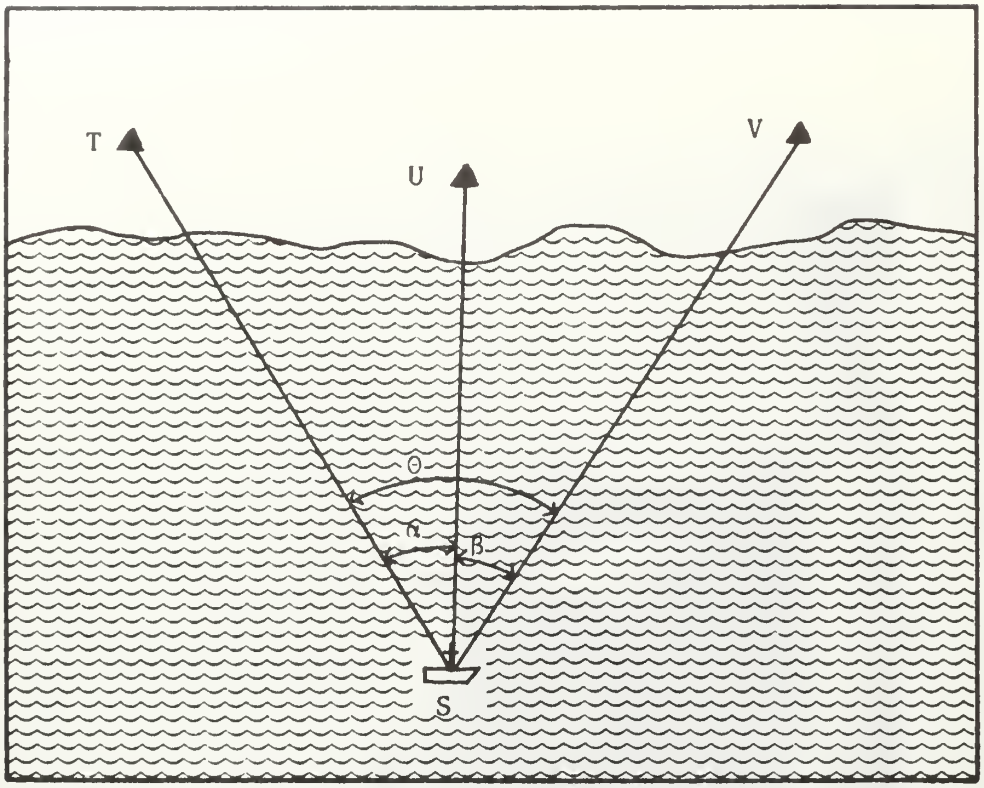

Figure 2.2. Location by two angles at the survey ship.

in Article 2.3.2.

The three-point resection locating method is advantageous from the standpoint that the entire operation takes place aboard the survey craft. Also, if a sufficient number of well-defined prominent objects of known location are visible along the coast, the task of establishing preliminary shore control is considerably reduced. A significant disadvantage, and it applies to all conventional visual positioning procedures, lies in the fact that they may be performed only in daylight hours and during periods of good visibility.

Another serious disadvantage of the resection method results from the fact that the ship is a continuously moving observation platform. In addition, the instruments are of low accuracy or have relatively slow readout capabilities. Hence, when greatest accuracy is required

Figure 2.3. Observing sextant angles to prominent points ashore from survey vessel.

the intersection method must be used.

(b) By directions from shore.

This method consists of determining the position of the survey craft by simultaneously observing intersecting directions to the ship from two or more control points ashore. The shore stations are normally determined in advance by triangulation, traverse or trilateration.

With the ship either moored or drifting slowly, the simultaneous directions are observed to it from the shore stations, denoted as T,

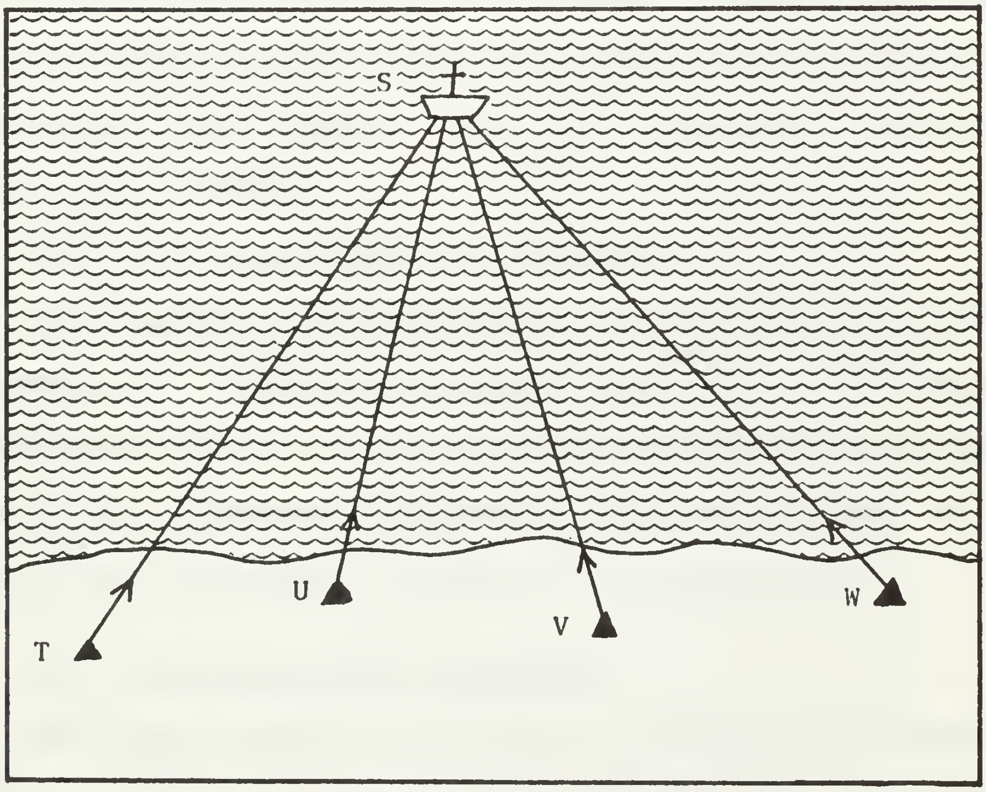

Figure 2.4. Location by directions from shore stations.

U, V and W in Figure 2.4, with either a theodolite or a continuously tracking azimuth instrument. A radio system is employed to coordinate the observation procedures and to relay direction data to the ship for

plotting.

The principal advantage of the intersection method is that observations are made from accurately determined points. For this reason and because the observing platform is stable, higher precision instruments may be employed. These factors result in a much higher accuracy in the ship position location than is possible with the resection method from the vessel. Another advantage of this method is that the preliminary task of setting out and erecting shore signals is avoided. Also, the problem of maintaining the vessel in a precise location in areas where currents are strong is eliminated. Disadvantages accrue from the separation of the survey party members and the associated problems which can occur with faulty radio communications between the ship and shore stations.

(c) By ship angles and shore directions.

This procedure, the simultaneous resection-intersection problem, is essentially a combination of methods (a) and (b). Since the observations taken from the shore stations are considerably more accurate than the ship information, this method serves primarily only as a check on the ship observations. In practice it is seldom used.

2.3.2 Resection Solution Procedures.

Three basic methods are available for determining the sextant fix position. They are all examples of the solution of the classic three-point resection problem. Using the notations in Figure 2.2, the problem may be stated as follows: given three shore points T, U and V and the values α and β of the observed sextant angles at the ship; to determine the position S of the ship. As mentioned in Article 2.3.1, the

Figure 2.5. Plotting sextant angle information aboard a survey vessel with a three-arm protractor for mechanical solution of the resection problem.

procedure most widely used in practice is the mechanical solution shown in Figure 2.5. Two others, the graphical and the analytical, are also available. However, for practical reasons, primarily the excessive time required, they are not usually employed in hydrographic survey operations.

(a) Mechanical solution.

In this procedure, the two horizontal sextant angles α and β are set into the three-arm protractor. The three arms of the protractor are next made to coincide with the three shore objects T, U and V which have been transferred to the plotting sheet. The center of the protractor then indicates the position S of the survey craft at the time of the observation. A small hole located at the center of the protractor permits the position of the fix to be marked on the plotting sheet.

(b) Graphical solution.

Several methods are available by which a graphical solution to the resection problem may be obtained. Three such constructions are described in (5). A representative procedure is outlined below. Referring to Figure 2.6, the lines between T and U and between U and V are first joined. At T and U the lines TO_1 and UO_1 respectively are set off from TU as indicated with angles equal to 90° minus the observed sextant angle α which was subtended at S between T and U. From the line UV in a similar manner, the lines UO_2 and VO_2 are set off at 90° less the sextant angle β. Two circles are then drawn, the first with center at O_1 and passing through T and U, and the second with center at O_2 and passing through U and V. The desired position of the ship is the intersection S of the two circles. That this is so may be shown by

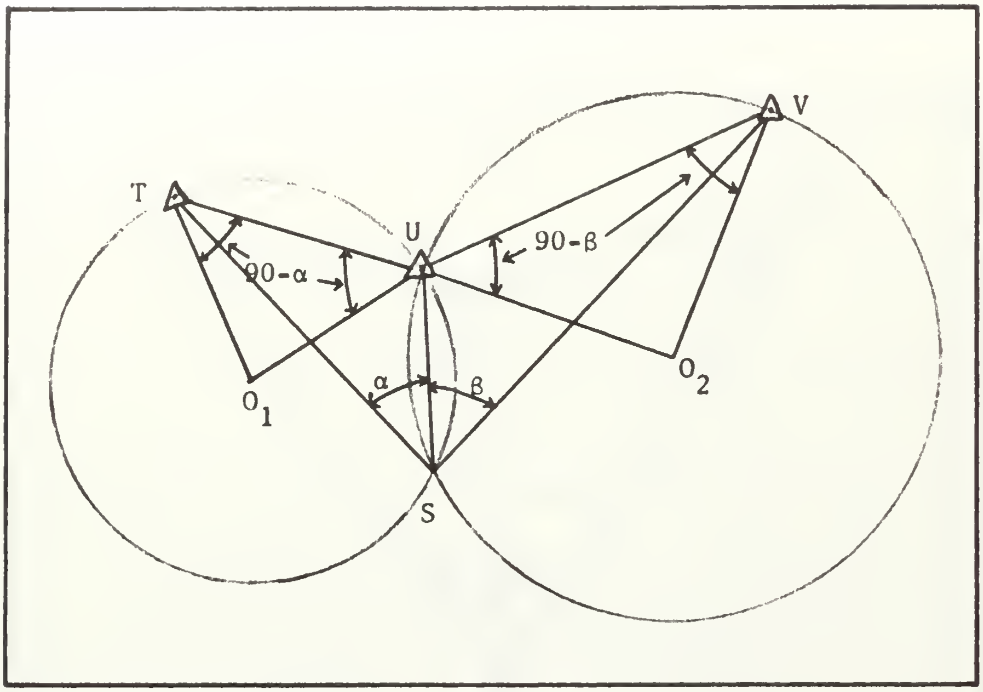

Figure 2.6. Graphical solution of the three-point problem.

the fact that the angle at S between T and U at the circumference of circle O_1 is equal to half the angle at O_1 between T and U on the same chord. But this angle is 2α. Thus the angle at S between T and U is α. In similar fashion it can be shown that the angle at S between U and V is β.

(c) Analytical solution.

Numerous procedures have been developed for the analytical solution of the three-point problem. A discussion of two such methods and an extensive theoretical and practical treatment of sextant positioning is given in (6). The following procedure is taken from (5). Referring to Figure 2.7, the lengths TU and UV and the angles α, β and γ are considered as the given quantities. Using the law of sines,

$$SU = \frac{TU \sin e}{\sin \alpha} = \frac{UV \sin f}{\sin \beta} \tag{2.1}$$

Now letting

$$d = e + f \tag{2.2}$$

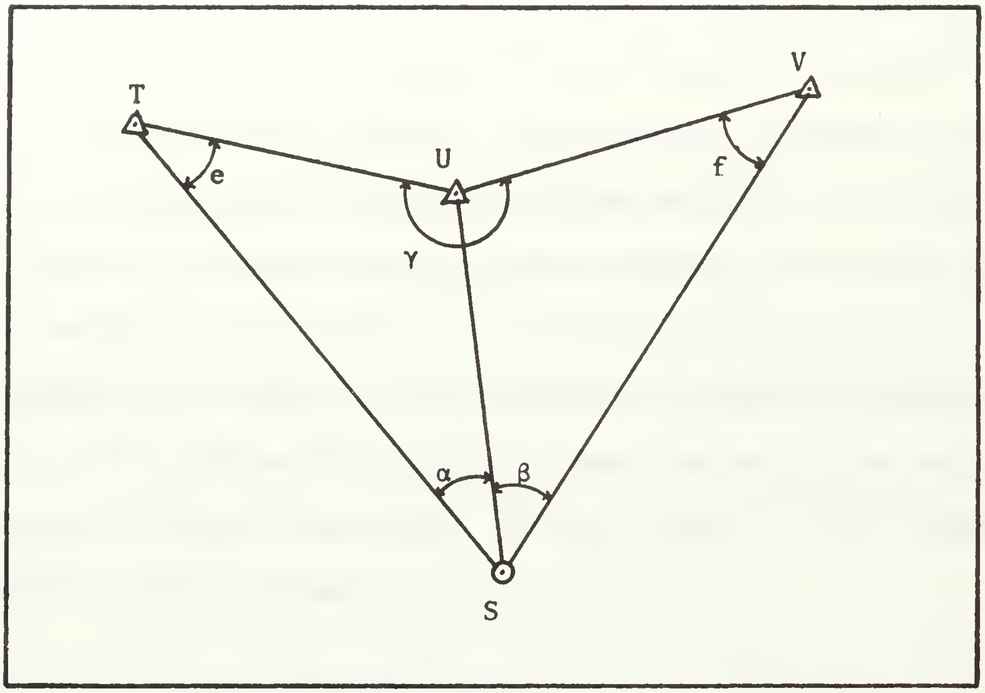

Figure 2.7. Analytical solution of the three-point problem.

equation (2.1) can be written as:

$$SU = \frac{TU \sin e}{\sin \alpha} = \frac{UV \sin(d-e)}{\sin \beta} \tag{2.3}$$

Using the relationship for the sine of the difference of two angles, the right side of (2.3) becomes:

$$\frac{TU \sin e}{\sin \alpha} = \frac{UV(\sin d \cos e - \cos d \sin e)}{\sin \beta} \tag{2.4}$$

Solving (2.4) for e and simplifying,

$$e = \cot^{-1}\left[\frac{TU \sin \beta}{UV \sin \alpha \sin d} + \cot d\right] \tag{2.5}$$

With e from (2.5), the angle f can be computed from (2.2) since d is a known quantity, given by:

$$d = 360^\circ - (\alpha + \beta + \gamma) \tag{2.6}$$

Finally, the distances SU, ST and SV are calculated using the law of sines.

2.3.3 Intersection Solution Procedures.

As in the case of the resection problem, numerous acceptable methods are available for the solution of the intersection problem. The graphical and mechanical procedures consist simply of extending directions or plotting angles from fixed or known positions. The intersections formed by the known direction lines determine the location of the desired points.

An analytical solution of the intersection problem is described in Chapter 5. The problem is explained in detail in conjunction with the associated adjustment computation. For this reason it is not treated separately in this chapter.

2.3.4 Equipment Characteristics.

(a) Hydrographic sextant.

The primary instrument employed in the visual methods of hydrographic surveying is the hand-held hydrographic sextant which is used to measure the horizontal angle between two shore objects. It has a maximum range of about 140° and is capable of measuring an angle to the nearest minute. The principle of the sextant is that the deviation of a ray of light reflected successively from two mirrors is twice the angle between them. One mirror, a large index glass, is mounted on an axis about which it can be rotated. The variable angle between the index glass and the fixed horizon glass mirror formed when the two objects sighted are brought into coincidence measures the angle subtended by the objects.

There are several sources of error in the sextant, some of which

may be adjusted for by the user and some which may not. OSBORN(2) gives a complete description of the various errors and how they may be minimized or removed. The four principal non-adjustable errors are summarized below.

(1) Prismatic error which occurs when the mirrors are not parallel.

(2) Graduation errors in the arc, micrometer drum and vernier of the sextant which are present when the instrument has been improperly constructed or calibrated.

(3) Centering error which results when the index arm is not pivoted at the exact center of curvature of the arc.

(4) Telescope error, resulting from resolving power, magnification and curvature of field characteristic limitations.

The four adjustable errors which together comprise the index error are:

(1) The error which results when the index mirror has not been adjusted so as to be perpendicular to the frame of the sextant.

(2) Side error which occurs when the horizon glass is not adjusted so as to be perpendicular to the frame of the sextant.

(3) Nonparallelism of the index mirror and the horizon glass when the index arm is set exactly to zero.

(4) Nonparallelism of the telescope and the frame of the sextant.

The determination of the errors of a sextant is normally based on a series of observations. With careful adjustments and proper handling, the sextant will remain in adjustment for a considerable length

of time. Checks are made, however, prior to and after each operation.

The greatest sources of error in sextant angle observations result from the shore objects being observed. This is particularly so when the shore points are buildings, water towers and natural objects which do not present a clearly defined narrow vertical line for sighting. Only with such objects as radio antennas and shore signals constructed by the survey party can maximum accuracy be attained. Poor contrast between the signal and the background region can also seriously degrade the accuracy of an observation. Another factor which normally far outweighs instrument limitations is the continuous and usually irregular movement of the survey vessel. This rolling and pitching can severely limit the observer's ability to obtain an accurate sighting. Indeed, observations must often be discarded and repeated when an observer is unable to make a proper sighting at the designated fix time.

Atmospheric conditions which cause haze, glare and similar phenomena; and the inability of the human eye to detect objects that subtend an angle smaller than a minute of arc also contribute to reduced accuracies.

It should be clear from the foregoing discussion that it is almost impossible to assign a meaningful standard accuracy value to a sextant angle observation. The conditions which exist during the survey must be analyzed and weighed in order to arrive at satisfactory values to be assigned for a particular project.

( ) Ship's gyro compass.

Compass bearings are much less preferable than sextant angle cuts in normal coastal hydrographic surveying operations as indicated in

Article 2.3.1. However, in the problem to be dealt with in Chapter 4, compass bearings must be taken to terrestrial objects for use as observed azimuths in the figural adjustment computations. For this reason, the error sources which influence and affect the resultant accuracy of a gyro compass bearing are briefly described.

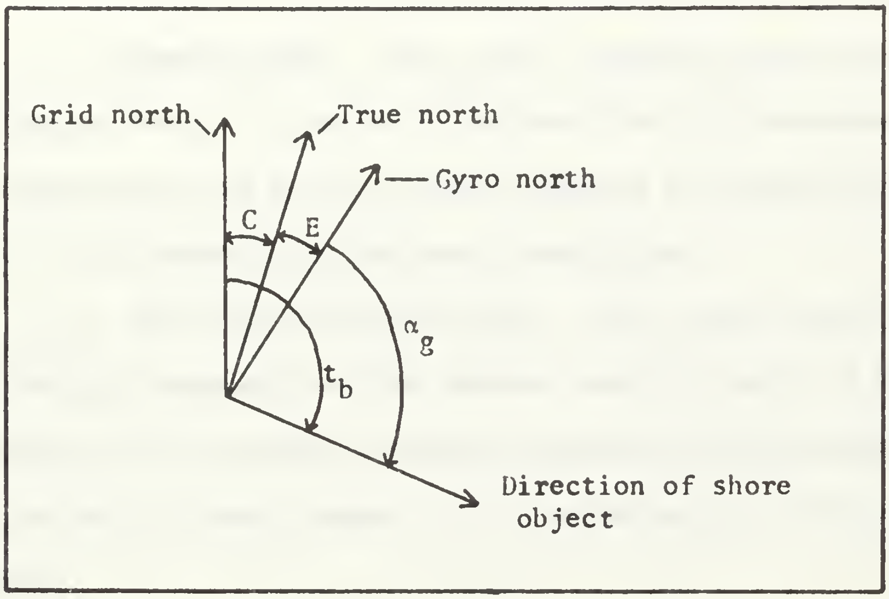

Figure 2.8. Gyro compass direction components.

Referring to Figure 2.8, C is the meridian convergence and E the gyro compass orientation error. The observed gyro compass bearing to the shore object is α_g, while t_b is the observed plane azimuth of the shore object referenced to the grid system being employed. Thus t_b may be expressed as follows:

$$t_b = \alpha_g + E + C \tag{2.7}$$

The gyro compass orientation error is the resultant of several systematic errors, certain of which are eliminated or compensated for in the construction of the compass and others which are minimized by adjustments made aboard ship. The five principal errors are enumerated

below. Detailed descriptions of these errors with procedures for eliminating or minimizing each of them are given in BOWDITCH(3).

(1) Speed Error. This error is introduced by the motion of the ship along its track. The direction of ship travel affects the magnitude of the error. It is corrected for in the design of the compass.

(2) Damping Error. This error, sometimes known as the ballistic damping error, is present in certain types of compasses. Its extent depends upon the method in which damping is accomplished in the compass. It is removed with suitable corrections.

(3) Ballistic Deflection Error. This error occurs because an accelerating force acts on the compass causing a surge of mercury from one part of the system to another each time the north-south component of the ship's speed changes. It is minimized with certain compass adjustments.

(4) Quadrantal Error. This error is caused by the swinging of the compass, an effect which varies according to the ship's heading. It is corrected for by adding weights to balance the compass so that the weight is the same in all directions from the center.

(5) Gimballing Error. This error results when compass directions are measured in a tilted plane. When the compass is tilted, the projection of its outer rim onto the horizontal is an ellipse. Thus the graduations are not equally spaced with respect to a circle. The best way to avoid this error is to read the compass only when it is horizontal or nearly so.

The residual gyro compass orientation error which remains after

all corrections and compensations have been applied, normally amounts to only about a tenth of a degree. It is applied equally to gyro bearings in all directions in order to obtain true bearings.

Since the gyro compass can be read only to tenths of degrees, it can be assumed for the determination of azimuth values that the plane azimuth t and the projected geodetic azimuth T are the same. This is permissable because the (t-T) correction seldom exceeds several seconds of arc.

A correction for the meridian convergence C must be computed and applied to convert true azimuths to grid azimuths if an X-Y coordinate system such as UTM is used for plotting. With a geographic latitude and longitude selected from the center of the project area plotting sheet, sufficient accuracy for computation of the difference between the true and grid azimuths is obtained with the following formula:

$$d = \Delta\lambda \sin \Phi \tag{2.8}$$

In this expression d is the difference in azimuths, $\Delta\lambda$ is the difference in longitude between the point and the central meridian and Φ is the latitude of the point. The sign of the correction depends upon the hemisphere and the side of the central meridian on which the computation point is located.

The considerations in determining the accuracy of a gyro bearing direction observation are essentially the same as previously discussed in the case of the sextant angle observation. However, since the compass reading is limited to tenths of degrees and because the uncertainty in determining the residual compass orientation error may amount to almost a tenth of a degree, the resultant direction observation

accuracy limiting factor is caused by the instrument rather than by external influences. A series of observations is taken with the compass to ascertain the reading accuracy. This value is then combined with the instrument accuracy determined for the individual compass to arrive at an overall observation accuracy.

2.3.5 Position Accuracy.

Much has been written concerning the position accuracies which can be obtained with sextant angle measurements. A foremost requirement for obtaining satisfactory results is that the figure formed by the shore control points and the ship should be of maximum strength. Certain general rules may be applied concerning the geometric strength of the resection figure. A fix will be weak in the case where the point to be located is near a circle which passes through the three known

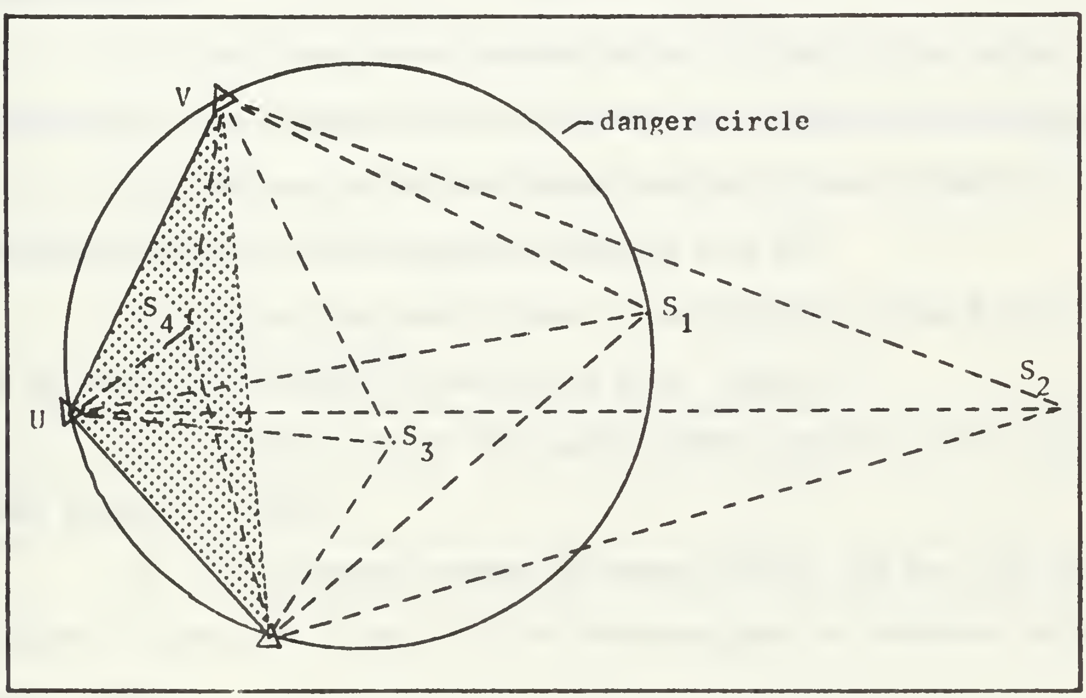

Figure 2.9. Geometric strength considerations in the resection problem.

points. Indeed, on this "danger circle" as it is known, the position (S_1) is indeterminate (see Figure 2.9). A fix will also lack strength if the new point (S_2) is so far from the fixed points that the fixed base lengths between known points are ineffective. Conversely, if the resected point (S_3) is near one of the known points it's position will be well determined. Any point (S_4) inside the shaded triangle formed by the three fixed points will be strongly determined.

With these and certain other less obvious criteria, the following rules have been formulated as the conditions for obtaining the best position when the resection principle is applied to the coastal hydrographic survey situation. The strongest fix occurs when, according to JEFFERS(7):

(1) The observer is inside the triangle formed by the three shore objects.

(2) The three shore objects are in a straight line or the center object lies between the ship and the line joining the other two.

(3) The sum of the two sextant angles is greater than 50° and preferably when neither angle is smaller than 30°.

(4) Two of the shore objects a considerable distance apart are in range and the angle to the third is at least 45°.

(5) At least one of the angles changes rapidly as the ship moves along it's track.

(6) The distance between the center object and the left and right-hand objects is longer than the distance from the observer to the center object.

A method devised for the solution of the three-point hydrographic

survey problem and the error of position that results from a known error in the sextant reading is described in detail by ALEXANDER(6).

2.4 ELECTRONIC POSITIONING.

Electronic control is the primary location determining method employed beyond the normal visibility from shore. Electronic control is also employed in coastal surveying during periods of low visibility and when adequate visual control is not available. Numerous electronic positioning systems are currently in use for hydrographic surveying control. Practically all of them are based upon either the circular (direct ranging) or hyperbolic method of line positioning. A few are composite systems, utilizing certain aspects of both basic methods. The principles and procedures for using the various methods are beyond the scope of this investigation. Numerous publications treat the various system in detail; among them LAURILA(8) and S.P. 39 of the International Hydrographic Bureau(9). For the purposes of this investigation only a brief summary of the basic characteristics and relative merits of the systems need be mentioned.

2.4.1 Circular Systems.

The circular or direct ranging methods depend for positioning determination on the measurement of the round trip travel time of radio waves from the ship's transmitter-receiver to accurately positioned shore stations. Families of concentric circles are produced as shown in Figure 2.10. The intersection of any two lines of position determines the position of the vessel.

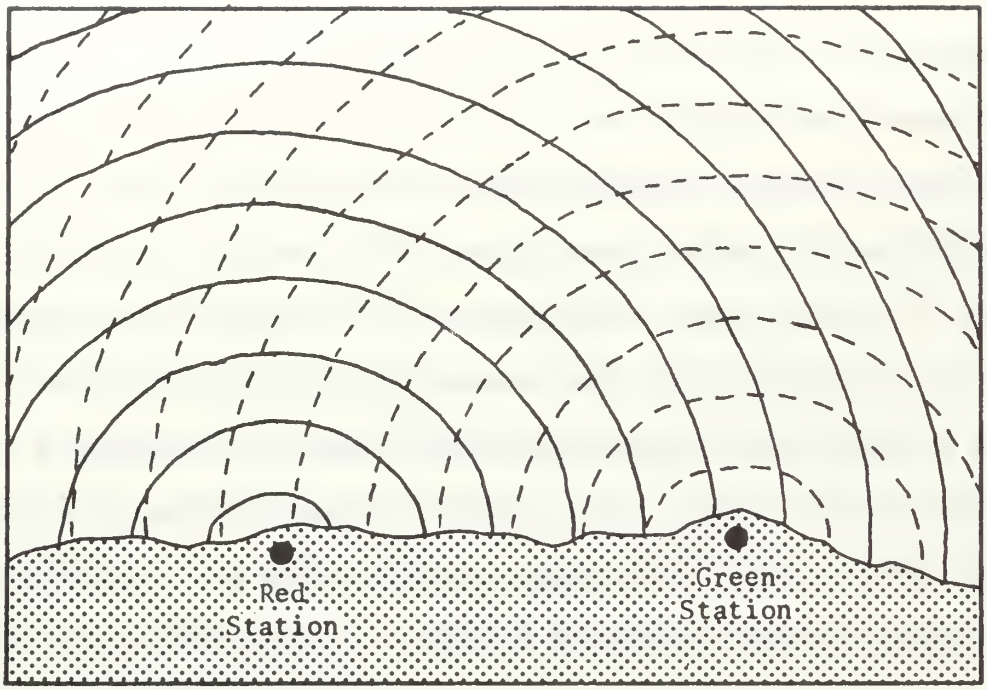

Figure 2.11. Circular electronic positioning system network.

Since such systems require radio transmission by the ship, the number of users is limited. Time-sharing procedures must be employed when more than a single ship is to use the same shore equipment and radio frequencies at any particular time. Ranges of such systems suitable for hydrographic surveying operations are generally limited to less than one hundred miles, due primarily to the high frequencies used. The all-weather capability of this and other electronic positioning systems renders them much more suitable for most surveying work than the visual methods.

The network geometry of the circular systems permits increased accuracy in positioning over larger areas in the grid compared with the hyperbolic systems because the spacing between curves of the same family remains constant. This characteristic is discussed in Article 2.4.4.

2.4.2 Hyperbolic Systems.

The hyperbolic systems depend upon the measurement of the time or phase difference of radio waves emitted from synchronized transmitters located at known positions ashore. The shipboard receiver determines the difference in arrival of the radio signal pulses. These difference measurements for each set of two transmitters cause families of hyperbolic lines of position to be generated with the transmitters at the foci. A minimum of two such systems are normally established as a unit, with a master transmitter acting as the control for two end or slave stations. The master station triggers the slave stations' pulses from it's own transmission. A typical hyperbolic network pattern is shown in Figure 2.11.

As with the circular systems, the intersection of two or more

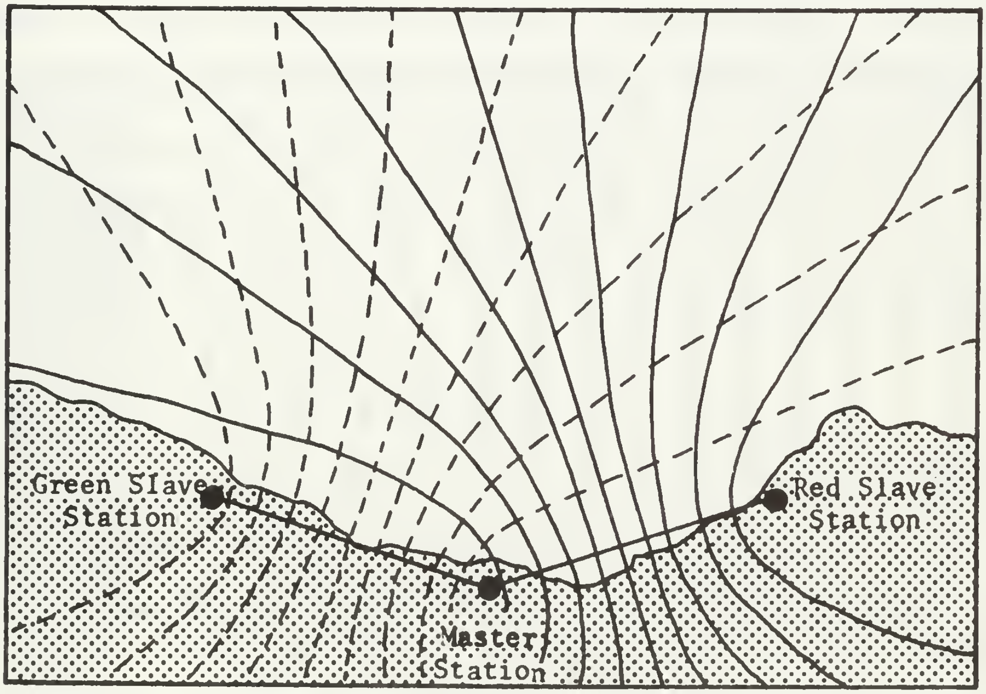

Figure 2.11. Hyperbolic electronic positioning system network.

lines of position determines the location of the ship. The accuracy of positioning varies considerably within the network, depending on the angle of intersection formed between the hyperbolic curves and the separation between curves of the same family at the location of the ship. Position accuracies and their determination are described in detail in Article 2.4.4.

Since the survey craft requires only a receiver, the number of users of hyperbolic systems is unlimited. The lower frequencies utilized in these systems permit greater ranges than with the circular methods.

2.4.3 Composite Systems.

Ranging and hyperbolic systems may be combined into composite positioning systems. Such systems are normally developed to improve the geometry of the electronic lattice. They are usually constructed

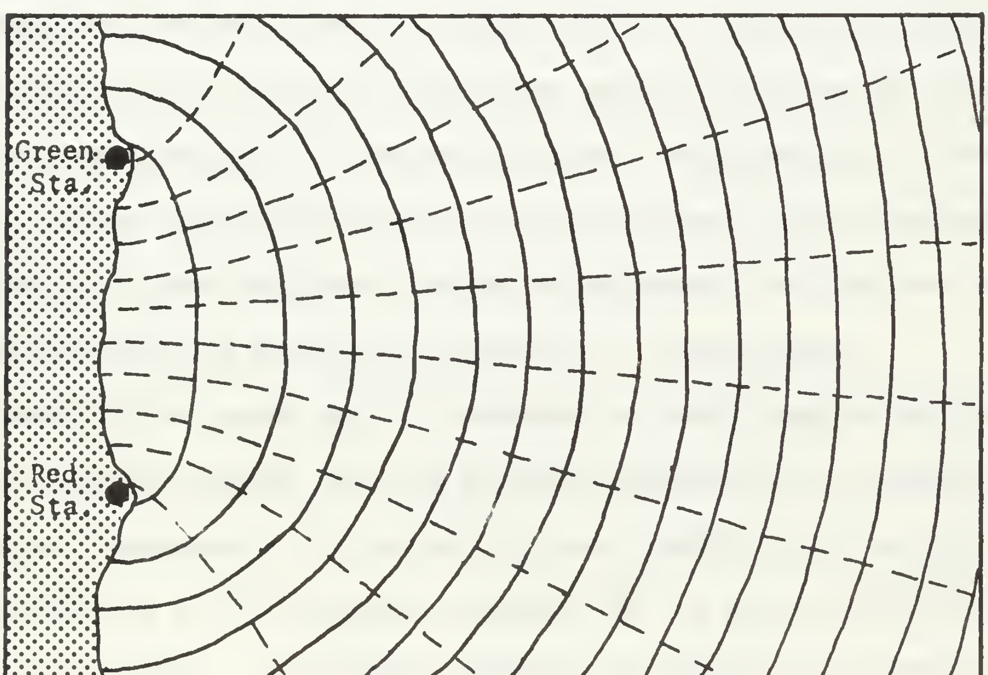

Figure 2.12. Composite Electronic Positioning System Network.

to satisfy a particular requirement, often when the area to be surveyed does not lend itself to the satisfactory installation of either a ranging or a hyperbolic network.

For example, one shore station acting as a master for a hyperbolic line of position may also generate a circular line of position. A second station acts only as the slave for the hyperbolic pair.

Another type of composite system combines elliptical and hyperbolic lines of position as shown in Figure 2.12. A principal advantage of this combination is that right angle intersections between families of lines are present throughout the network.

2.4.4 Position Accuracy.

The position accuracy obtained with any electronic system may be thought of as composed of two separate parts: repeatability and predictability. Repeatability is defined as the reliability with which the electronic method allows the survey vessel to return to a particular location on the surface of the ocean specified only by the lines of position generated by the electronic system. Predictability is the measure of the reliability with which the electronic method defines the location of a given position in terms of geographic latitude and longitude rather than the electronic hyperbolic or range values.

Repeatability error may be considered as being composed of those errors which are present when the system is operated in an idealized situation. Components of repeatability are dependent only upon the characteristics of the equipment hardware and the precision with which it can be operated. A principal component of repeatability error is

the uncertainty associated with geometric properties of the system network, such as line spacing and curve intersection angles. Also included are uncertainties caused by instrument construction characteristics which limit equipment measurement accuracy. Another portion of repeatability error arises from the inaccuracy with which an operator interprets instrument results.

Predictability error results when environmental variations and changes which occur with the passage of time are imposed on the system. Such uncertainties arise when the system is operated in a real situation. The principal causes of predictability error are variations in propagation velocity and atmospheric refraction.

Both repeatability and predictability errors are composed of systematic and random influences. Repeatability contains a substantial amount of systematic variation. Therefore, with careful equipment calibration and analysis of the system, repeatability errors can be fairly well determined and allowed for. Such is not, however, the case at the present time with predictability errors. They are difficult to isolate and remove. Empirical formulas have been devised for use in attempting to correct for atmospheric condition variations. However, the usefulness of such formulas remains questionable(10).

As mentioned above, repeatability error can be satisfactorily determined because it depends primarily upon known factors. The development of the expression for the repeatability error of a position which follows, applies equally well to a ranging, hyperbolic or composite positioning system. In Figure 2.13, S is the apparent location of an electronically determined ship position in the geometric network of the

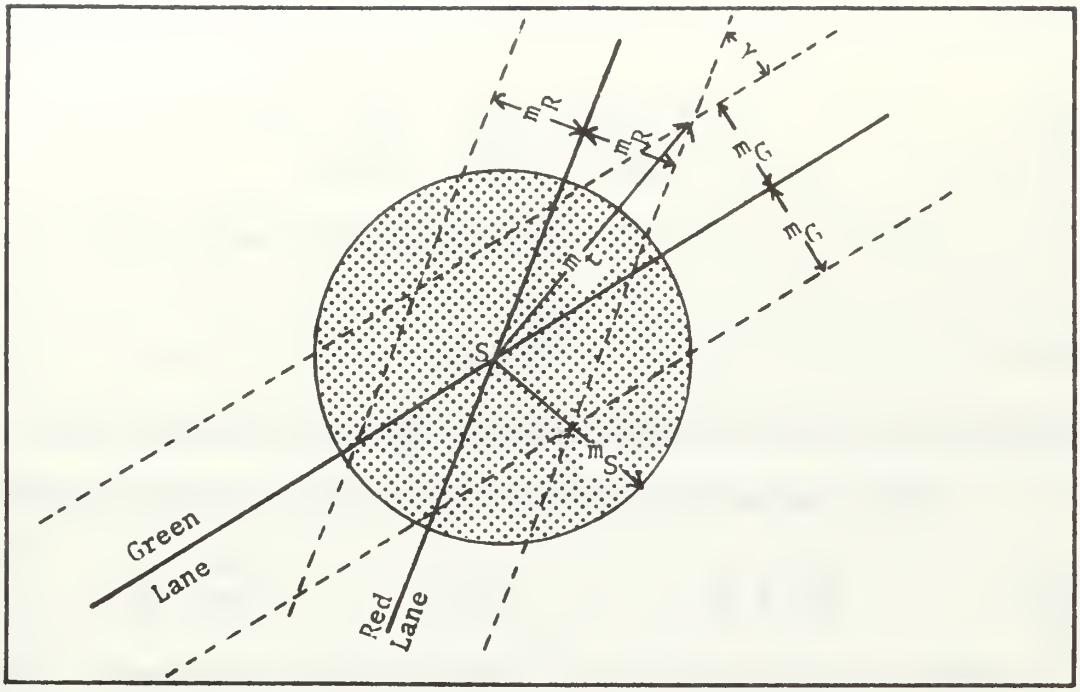

Figure 2.13. Repeatability error circle for electronic positioning systems.

system. The angle of intersection of the lines of position, be they hyperbolas or circles or one of each, is designated as γ. The standard errors of the displacement of the two lines of position, denoted as RED and GREEN, are m_R and m_G.

True curves of constant probability would be ellipses centered at S as shown in Figure 2.14. Since for most systems these curves are somewhat difficult to compute and construct, for this general development a more suitable figure, the circle, is substituted. The actual position of the fix may then be said to lie anywhere within the circle centered at S with radius m_S with a 68% probability. In this simplified form, the radius m_s of the error circle is considered to be the semi-major axis of the error ellipse.

The position error of a single observation m_t in terms of m_G, m_R

and the angle γ is expressed as follows:

$$m_t^2 = \frac{m_G^2}{\sin^2\gamma} + \frac{m_R^2}{\sin^2\gamma} + \frac{2m_G m_R \cos\gamma}{\sin^2\gamma} \tag{2.9}$$

Hence, with a total of n observations:

$$m_S^2 = \frac{\Sigma m_t^2}{n} \tag{2.10}$$

Now if q_G and q_R are the standard errors of each line of position obtained by an error analysis of the system concerned, then:

$$q_G^2 = \frac{m_G^2}{n} \quad \text{and} \quad q_R^2 = \frac{m_R^2}{n} \tag{2.11}$$

Next, a correlation factor R which is one if every radius vector is common to at least two pairs and zero if none is used in more than one pair is introduced as:

$$R = \frac{m_G\ m_R}{n\ q_G\ q_R} \tag{2.12}$$

Substituting (2.9) into (2.10) gives:

$$m_S^2 = \frac{1}{n \sin^2\gamma}\left[\Sigma m_G^2 + \Sigma m_R^2 + 2\Sigma m_G m_R \cos\gamma\right] \tag{2.13}$$

Solving (2.11) and (2.12) for the displacements m_G and m_R and substituting into (2.13) yields:

$$m_S^2 = \frac{1}{n \sin^2\gamma}\left[nq_G^2 + nq_R^2 + 2nRq_G q_R \cos\gamma\right] \tag{2.14}$$

Simplifying (2.14), the standard error of a position, the radius of the error circle in Figure 2.13, is:

$$m_S = \operatorname{cosec}\gamma\sqrt{q_G^2 + q_R^2 + 2Rq_G q_R \cos\gamma} \tag{2.15}$$

Finally, the weight to be assigned to each position would be computed as:

$$p_S = \frac{1}{m_S^2} \tag{2.16}$$

In the case of a hyperbolic system, the error components in the directions of the rectangular coordinate axes can easily be obtained. Instead of substituting the standard error circle for the ellipse, the erroneous displacements of the point S along the RED and GREEN position lines are computed. These errors, denoted as e_R and e_G respectively in

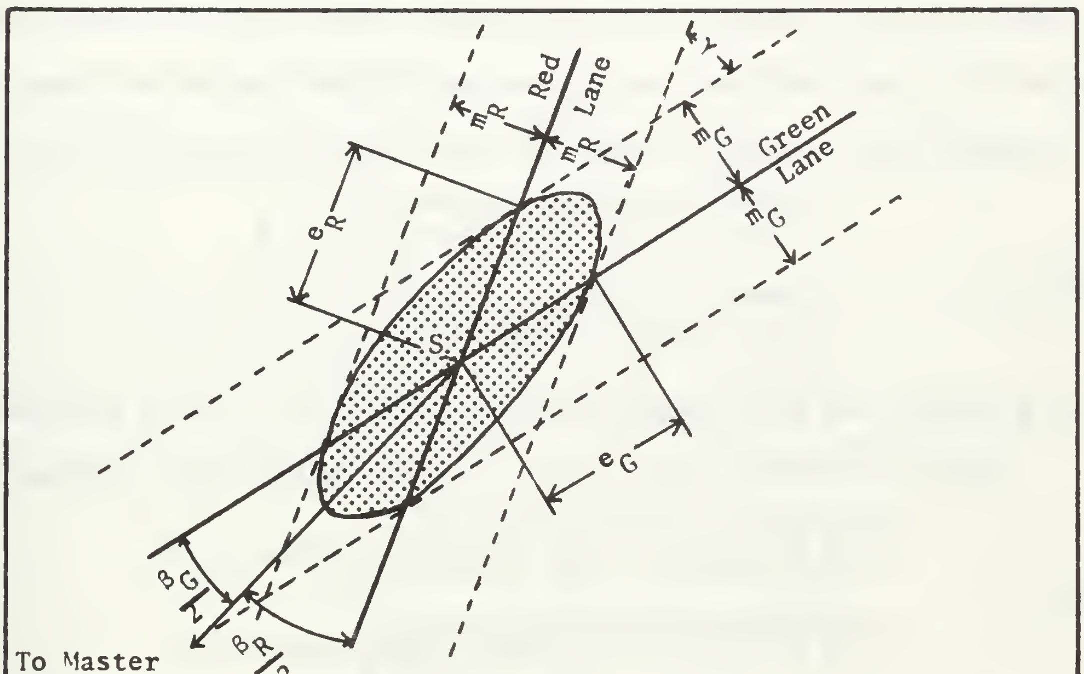

Figure 2.14. Repeatability error ellipse for electronic positioning systems.

Figure 2.14, are the conjugate axes of the error ellipse. They are computed from the geometry of the figure as:

$$e_R = \frac{m_R}{\sin\gamma} \qquad \text{and} \qquad e_G = \frac{m_G}{\sin\gamma} \tag{2.17}$$

It can also be shown, according to LAURILA(8), that:

$$m_R = q_R \, l'_R \frac{1}{\sin \frac{\beta_R}{2}} \qquad \text{and} \qquad m_G = q_G \, l'_G \frac{1}{\sin \frac{\beta_G}{2}} \tag{2.18}$$

where l'_R and l'_G are the lanewidths on the RED and GREEN baselines and q_R and q_G are the electronic errors in the RED and GREEN position curves as described earlier in the development. Both l' and q values are constant for a given system.

Now if the error components e_R and e_G are considered to be mutually independent, the following expressions may be written for their projections in the X and Y directions where α_R and α_G are the angles between the RED and GREEN lines respectively and the Y-axis of the cartesian grid system on which the electronic lattice is superimposed.

$$\begin{aligned} m_X &= \sqrt{(e_R \sin \alpha_G)^2 + (e_G \sin \alpha_R)^2} \\ m_Y &= \sqrt{(e_R \cos \alpha_G)^2 + (e_G \cos \alpha_R)^2} \end{aligned} \tag{2.19}$$

Combining (2.17), (2.18) and (2.19) and simplifying the resulting expressions, the standard errors in the X and Y directions become:

$$\begin{aligned} m_X &= \operatorname{cosec} \gamma \sqrt{(q_R l_R \sin \alpha_G)^2 + (q_G l_G \sin \alpha_R)^2} \\ m_Y &= \operatorname{cosec} \gamma \sqrt{(q_R l_R \cos \alpha_G)^2 + (q_G l_G \cos \alpha_R)^2} \end{aligned} \tag{2.20}$$

This development assumes that the error components for the two lines of position are mutually independent. No information could be found either from a review of the literature or in discussions with personnel at the U.S. Naval Oceanographic Office concerning estimation of the dependence between survey accuracy determinations at intersections of lines of two or more families of circular or hyperbolic curves.

Allowing for the future possibility that estimates of covariance may be determined, the program for the numerical example calculation in Section 4.6 has been designed to accept and utilize such information.

The following table, extracted from BIGELOW(10), is intended to

Table 1. Electronic Positioning System Accuracies

System	Type	Range (naut.mi.)	Accuracy(1) (feet)	Accuracy(2) (feet)
Loran-C	hyperbolic	1200	50	1200
Lorac-A	hyperbolic	200	15	400
Decca Survey	hyperbolic	200	25	300
Hi-Fix	hyperbolic	40	25	150
Lambda	circular	425	25	250
DM Raydist	circular	200	12	100
Hi-Fix	circular	30	12	100
Raydist	composite	75	15	150

(1) For best part of service area under excellent conditions.
(2) At column three range under normal conditions.

give an indication of the accuracies (repeatability) which can be expected with the electronic positioning systems now in use for hydrographic surveying. These systems are the ones deemed most suitable for employment with the application developed in Chapter 4. Of significance is the generally greater loss of accuracy in the hyperbolic systems compared with the direct ranging systems at the maximum useful ranges.

2.4.5 Offshore Transmitter Platforms.

In conventional surveys, station sites for electronic positioning system transmitters are placed at known points ashore. These shore

points are located at geodetic network stations or are connected to them by triangulation, trilateration or electronic traversing.

In the problem dealt with in Chapter 4, however, the use of land-based station sites is precluded in the vicinity of the offshore survey. This means that two alternatives are available in so far as the use of an electronic positioning system is concerned. First, a long range system such as Loran-C could be utilized with a corresponding decrease in accuracy as compared with the short range systems normally employed in coastal hydrographic survey work. The second choice would consist of establishing the transmitters on floating platforms anchored offshore, whose positions in turn could be accurately determined. The latter of these two alternatives is treated in this discussion.

The U.S. Naval Oceanographic Office has developed such a technique and has demonstrated its feasibility for the operation of short range electronic systems along inaccessible coastal regions(11). For both the direct ranging systems with two stations and hyperbolic methods with three, the use of floating platforms has been demonstrated successfully. In the former case the platforms are located to provide baseline distances of up to twenty-five miles and in the latter lines of up to sixty or more miles. In each case, of course, transmitter separation distances are dependent upon the specific positioning system being employed and the environment of the survey area.

For such an offshore installation the system transmitters are located in small boats, barges or buoys positioned five to ten miles from shore. When boats and barges are used, they are anchored bow and stern to provide for maximum positioning stability. Also, to ensure that the

floating platform is maintained in its proper position during the period of the survey, reference buoys designed to permit only minimum horizontal movement are located and emplanted near the floating platform for position recovery during the survey operation.

The offshore platforms are positioned by ranging or trilateration methods from known positions. Of particular interest to this investigation is the method used when direct occupation of coastal points is impossible. In this situation a helicopter with an airborne positioning system receiver hovers directly over or as near the shore point as practicable. Range readings are taken and are repeated by making a number of passes over the point to allow computation of as accurate a distance as possible when the individual readings are later averaged. Another means employed, when hovering is impossible, is to fly over the point in a fixed wing aircraft at various widely separated headings, preferably about 90° apart. With a minimum of four such passes, on each of which the range value is recorded when passing over the known point, acceptable mean range values can be obtained.

2.5 OTHER POSITIONING SYSTEMS.

In addition to the visual and electronic methods, there are a number of other systems which can be used for the control of hydrographic surveys. Most are still in the experimental stage of development as concerns coastal hydrography applications. However, with the rapid technological improvements being made in these and related systems, it is not inconceivable that in the near future any or all of them may prove capable of supplying the necessary position accuracy required of

the ship-based problem treated in Chapter 4. For this reason certain of these other systems are briefly summarized. The capabilities and limitations as concerns the use of these systems in coastal hydrography are also presented. These characteristics are included to indicate why and how such systems could either improve or possibly degrade the results, compared with information obtained by the conventional visual and electronic methods now in use. Of the three methods described, the one which appears to show the greatest promise as a locating device for coastal surveying is the laser system.

2.5.1 Photogrammetric Methods.

Panoramic terrestrial photography taken from the survey ship has been utilized on a limited basis in an effort to supplement the usual coastal hydrographic survey information. Recently an attempt was made to integrate such photography of the coast with an automated survey system. In this test, camera circuitry caused interference in digitizing depth data and skips in data recording. Problems were also encountered in processing and analyzing the photography(12). This last mentioned deficiency in interpretation of the photography has been the greatest cause of problems with such systems. Poor contrast in the coastal areas with the resultant difficulty in accurately locating the shore targets has thus far limited the usefulness of photography in yielding the necessary object position information with which to reconstruct the survey situation.

A second photogrammetric method used experimentally to position a survey ship at sea involves an application of photogrammetric satellite

triangulation. The survey vessel is equipped with a precision satellite camera mounted so as to isolate it from the ship's motion. A gyrocompass is used to align the leveled camera with geographic north. The camera thus oriented is then moved to point in the predicted direction of the satellite as it passes overhead.

At least two other such cameras are mounted at known locations ashore and are also pointed in the predicted direction of the satellite at the time of its passage. The shore cameras are precalibrated with synchronized exposures to obtain their orientation. The photographs taken of the satellite from the shipboard camera are used to determine its orientation directly from the stars in the background(13). A synchronized timing system permits all cameras to photograph the satellite simultaneously for the purpose of determining the ship position.

The system is considered capable of achieving positions within 50 to 100 feet under normal conditions. An obvious disadvantage is that good visibility is required. Also, the data processing to obtain the position cannot yet be performed on a real-time basis since a portion of the data is at the shore stations. The lack of a continuous positioning capability, because of the elapsed time between satellite passages, also limits its applicability to survey operations. This deficiency and the possibilities for overcoming it are discussed in Article 2.5.3.

2.5.2 Lasers.

Considerable effort has been expended in the development of a laser theodolite for precise ship positioning in coastal hydrographic

survey operations(14). At the present time such laser systems are still in the experimental stage. However, they show considerable promise of being adopted on an operational basis with additional research and improvement.

The laser systems are to be used for both of the usual visual positioning methods, resection from the ship and intersection from shore stations. The relative merits of the two alternative methods are essentially those previously described in conjunction with the conventional visual systems. The primary advantage of the ship-based system is the single location of the entire positioning operation. The shore-based problem relies for its greatest strength on the stable platform and the precise location of the known terrestrial stations from which the observations are made. For these reasons the intersection solution is again more accurate than the resection from the ship.

It appears that the ship system could be readily adapted to the application described in this investigation since, in a hostile environment, a landmark with good reflectivity may be used as the shore target in place of the usual retrodirective target. For the shore-based problem, observations are taken from known stations to targets mounted on the ship.

With both systems, range as well as the usual direction information is obtained. With relative bearings to a tenth of a degree and line of sight ranges accurate to ten feet, the laser method could considerably strengthen solutions obtained through the adjustment computations developed in the Chapter 4 problem.

Although the laser beam is affected by atmospheric refraction, the

use of dispersion techniques and repetition of readings can substantially reduce the uncertainty of observational measurements. Compared with conventional visual systems, the laser method has the added advantage of being able to operate in fog, smoke and haze; conditions which render the usual visual methods impossible.

2.5.3 Satellite Systems.

Doppler satellite navigation systems are now capable of determining reliable positions at sea with an accuracy of less than 300 yards. Utilizing two receivers, the accuracy with these systems can be improved to less than 30 yards(15). To obtain a position, the satellite must be observed over a twelve minute period during its passage.

The lack of a continuous position determination capability is a distinct disadvantage of satellite systems as applied to hydrographic operations. Intervals of almost two hours, the period of an orbit, to more than three hours when the satellite passes within about ten degrees of the observers zenith, are inherent with such a system(16).

In order to obtain positions on a continuous basis another accurate system, such as an inertial navigation method capable of giving continuous velocity and heading of the ship, must be integrated with the satellite method. While satellite systems will provide satisfactory results for certain types of ocean survey operations, their application to coastal hydrography is considered minimal without significant improvements in overcoming the limitations enumerated above.

3. AUTOMATION IN HYDROGRAPHY

3.1 GENERAL.

The large-scale use of computers and automated procedures in hydrographic surveying operations is now being implemented by organizations such as the U.S. Naval Oceanographic Office and the U.S. Coast and Geodetic Survey. Hydrographic survey ships are being equipped with the necessary systems and equipment to allow depth, gravity, magnetic and positioning data to be acquired and processed automatically.

With the use of on-line computers, it will be possible to process the survey data and produce smooth sheets containing the hydrographic data on a real-time basis shortly after the data has been collected. From the smooth sheets, field charts will be produced within a matter of hours. This feature will be especially useful in areas of military operations where charts are required in the minimum length of time.

Since the use of computers aboard survey ships for the collection and processing of hydrographic data is now becoming a reality, the adjustment procedures developed in this investigation have been designed for such processing techniques and are programmed in the Fortran IV programming language.

3.2 AN AUTOMATED SURVEY SYSTEM.

In order to more clearly illustrate the type of an automated survey system to which the adjustment procedures developed in this investigation could be applied, the HYDRA System to be employed by the U.S.

Naval Oceanographic Office is described briefly. For a complete explanation of the system, see SPINNING et al (12). In Section 3.3, modifications to this system which would be required in order to permit acquisition and processing of the data required to solve the Chapter 4 problem are discussed.

The HYDRA System was developed principally for military area survey applications, also the primary area intended for the ship-centered adjustment procedure developed in Chapter 4. As such, this particular system is considered to be uniquely suitable as the method by which the problem could be implemented.

3.2.1 Data Acquisition.

The HYDRA Survey System basically performs three distinct functions: position fixing, depth acquisition and data recording. For position fixing, an electronic positioning system is employed. A digitizer unit is utilized to translate the output from the shipboard receiver into computer language, i.e. to convert lane values into digitized form.

A digital depth finding system consisting of an echo sounder, transducers and a depth digitizer is used to acquire sounding data. As with the positioning system, the digitizer forms the link between depth acquisition and data recording. The digitizer converts the traveling time between transmission pulse and bottom echo, measured by the echo sounder, into a digital output value.

The digitized position and depth data is correlated and sequenced by a digital control unit before being recorded on magnetic tape. The

information in both digitizers is interrogated and stored together with an internally generated time signal. The information is then sequentially scanned and recorded on magnetic tape in its proper location.

3.2.2 Data Processing.

The HYDRA Automated Data Processing System performs two separate functions: data processing and plotting. First, analyses are performed on the data collected. Errors in the data and weak areas such as locations where insufficient information has been collected are determined in the edit procedure. The second step entails plotting the ship's track by means of the digital plotter in combination with the data acquisition system. The third part of the process consists of correlating the depth information with the positioning data and analyzing the results to produce an optimum smooth sheet plot of the sounding data, to include the plotting of depth contours if desired. A paper tape is generated by the processor for use in plotting the results. This plotter tape, when fed into an off-line digital plotter, causes a smooth sheet containing the hydrographic data to be produced.

3.3 APPLICATIONS AND MODIFICATIONS.

The HYDRA Survey System was developed to satisfy the requirements for river surveys in military warfare regions. With modifications to allow the acceptance of positioning information from various other location determining systems such as satellite and visual, the use of this automated procedure could be extended to other areas of coastal hydrographic surveying and also to ocean surveying applications.

Two basic modifications would be necessary for the HYDRA System to render it capable of incorporating the procedures of the first problem dealt with in this investigation, the self-contained system aboard the survey ship. First, provision would have to be made for reading in the visual data; sextant angles and gyro compass bearings; and correlating it with the depth and electronic positioning information. Procedures have already been proposed for the integration of gyro information into automated acquisition procedures(17). In order to incorporate sextant angle information, a punched card procedure could easily be developed to introduce the angular and associated time information to the digital control unit for integration and sequencing with the other input data.

As a second modification, or more properly an addition, a program to perform the required adjustment calculations would need to be incorporated in the processor library to enable the required computations to be accomplished by the computer. Certain other minor modifications would also be necessary in order to convert the input data into a form suitable to allow the adjustment calculations to be performed.

4. DETERMINATION OF SHORE AND SHIP POSITIONS FROM VISUAL AND ELECTRONIC MEASUREMENTS AT SEA

4.1 GENERAL.

The approach to be taken in this problem utilizes the principles and procedures previously described for the ordinary coastal hydrographic survey. However, it varies considerably in that portions of ordinarily known information and available data are now considered as unobtainable for various reasons. With these limitations imposed, the remaining tools at the disposal of the hydrographer, including high-speed computer processing, are utilized to fill in the missing information and arrive at the desired positional results for the survey.

With this lack of certain usually available data, the accuracy of the ultimately determined positions will necessarily be somewhat less than had all of the supporting information been present. However, in operational situations it often becomes imperative to sacrifice a certain degree of accuracy in order to allow the required mission to be accomplished in an expeditious manner.

The situation to be dealt with in this instance is best applied along a stretch of inaccessible coastline. In this area a requirement exists for a rapid and fairly accurate determination of the positions of prominent objects ashore and water depths along the coast. As an example of such a situation the following military operation and the associated hydrographic requirements are outlined. Water depth information and landmark locations are needed along the coast of a hostile

nation so that combatant naval vessels may navigate safely and fix their positions visually in order to provide accurate offshore gunfire support for destruction of enemy fortifications. Accurate inshore soundings are also required to facilitate and insure the navigational safety of anticipated amphibious landings.

The only available nautical charts of the area are old and incomplete. They do not contain sufficient information to enable the navigator to fix his position visually and are practically devoid of sounding information. They include only a sparse few topographic features which could be used as navigation landmarks. The location of all charted information is questionable owing to the poor accuracies of the original surveys. An additional restriction imposed in this situation is that the area cannot be overflown for low level reconnaissance photography for use in planning and executing the survey.

Adjustment computation procedures will be applied to the observed data in order to arrive at the most likely values for the unknown quantities in the problem. Estimations of accuracies for all adjusted values will also be determined.

With sufficient shore control, the mechanical solution described in Article 2.3.2 normally provides adequate positions for the placement of sounding data on nautical charts. However, with little or no known terrestrial position information this conventional procedure cannot be applied. Also, even with satisfactory visual control ashore, the conventional sextant resection method may not yield results accurate enough to fulfill the most stringent offshore positioning requirements. For these reasons the adjustment procedure which follows has been

developed for application to coastal survey situations. Although the use of adjustment computations has seldom been applied in coastal hydrography, procedures have been devised for certain aspects of ocean surveying(18) and geodetic positioning at sea away from the coast(19).

A planimetric approach is taken to this problem since all visual observations, both sextant angles and compass bearings, are normally measured between objects in approximately the same horizontal plane as the observer aboard the survey vessel. When it becomes necessary to measure a sextant angle between two points with a significant difference in elevation, the angle is corrected before being used in plotting or computations by a graphical method based on the formula:

$$\cos \alpha = \frac{\cos O}{\cos h} \tag{4.1}$$

In this expression α is the horizontal or computed angle, O is the observed inclined angle and h is the angular elevation of the elevated object from the point of observation. The angular elevation h is normally determined by a sextant observation from the survey vessel.

4.2 PROBLEM OBSERVATION DATA.

It is assumed that an electronic positioning system has been established so as to give suitable coverage in the coastal area to be surveyed. Transmitter sites for the electronic system are established outside the inaccessible area, either in neighboring friendly territory or, if this is not feasible, aboard floating platforms moored at offshore locations as described in Article 2.4.5.

The survey ship, as it proceeds along the coast at a distance of

two to five miles from shore, obtains the usual sextant angles to prominent terrestrial objects. If the shore points are at elevations considerably above the height of the observer aboard the ship, the angles are reduced with (4.1) before being entered into the adjustment computation. This preliminary reduction of inclined angles, when necessary, allows the problem to be dealt with planimetrically in all cases.

Each time sextant observations are made, a gyro compass bearing is taken to the extreme object in the direction closest to the ship's track. This particular point is chosen since, in most instances, its bearing from the ship will be changing at a slower rate of speed than will bearings of the other shore points. A principal reason that only one such direction is observed for each ship fix position is that normally only a single gyro compass repeater is available for making such a reading.

As the visual data is being obtained, the electronic positioning system coordinates of the ship are recorded at each designated fix time. Depth information is recorded continuously during the operation. Any other desired information such as gravimetric determinations are also made simultaneously with the visual observations. Times are noted for all data readings to permit later correlation of the information.

Referring to Figure 4.1, the points S_1 through S_4 are successive positions of the survey vessel proceeding along the coast. The points T, U and V represent prominent objects along the shoreline such as towers, buildings and natural features between which the sextant angles α_i and β_i are observed. All compass bearings t_i are taken to

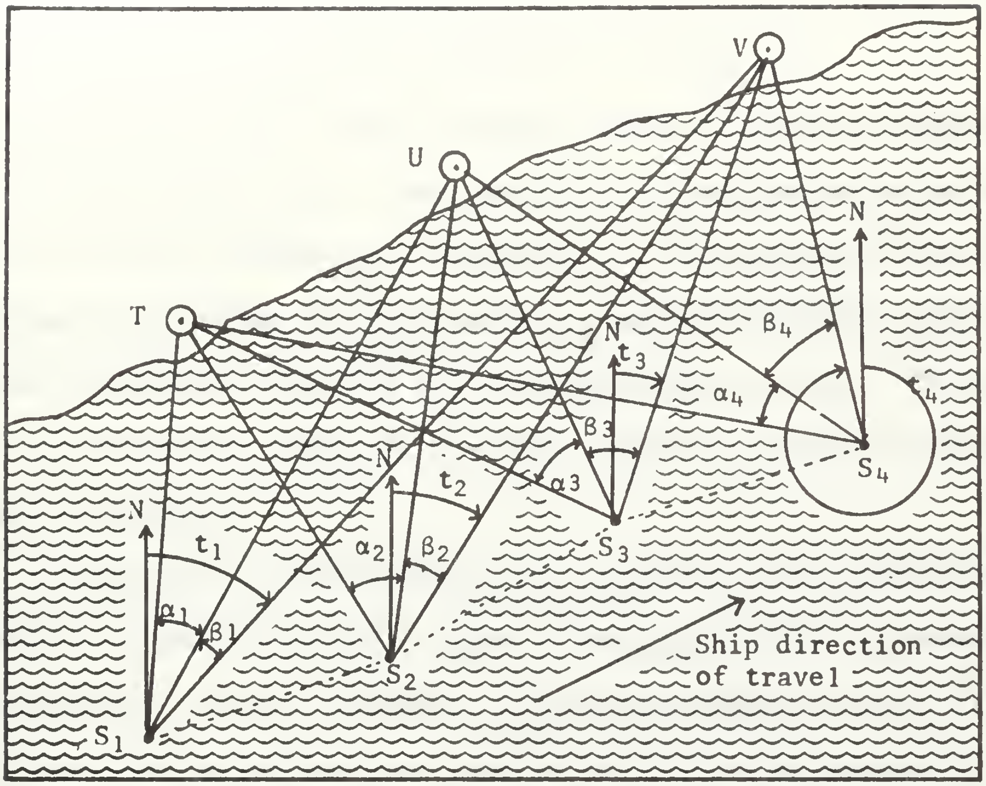

Figure 4.1. Coastal hydrographic survey network for ship observations.

terrestrial object V.

4.3 <u>ADJUSTMENT PROCEDURE</u>.

The generalized least squares adjustment method with parameters treated as observables is utilized for the solution of the problem. The matrix notations and adjustment procedure are according to UOTILA(20). Owing to the linear relationships which exist among the observed quantities and the unknown parameters, and the method used for computing approximations for the parameters, a satisfactory solution should normally be achieved without the necessity of recycling the adjustment. For completeness of the derivation, however, at the end of the general adjustment procedure described in this section and

in the detailed equation formations of Section 4.4, the modifications necessary for recycling the adjustment are explained.

Sextant angles and compass bearings taken on the ship compose one set of observations, $L1_b$. These observations are considered as belonging to the first mathematical structure F_1. The adjusted values of the observable quantities are designated $L1_a$ and the adjusted values of the unknown parameters, the ship and shore positions, are X_a. There is only one observable quantity, a sextant angle or a compass direction, in any F_1 equation.

A second set of observations, $L2_b$, consists of the successive positions of the survey ship, taken as belonging to the mathematical structure F_2. In this structure, $L2_a$ are the adjusted values of the ship positions, which are also some of the unknown parameters X_a.

The two mathematical structures may be written as:

$$\begin{aligned} F_1(L1_a, X_a) &= 0 \\ F_2(L2_a, X_a) &= 0 \end{aligned} \tag{4.2}$$

Through the Taylor series development, (4.2) can be expressed as follows:

$$\frac{\partial F_1}{\partial L1_a} V_1 + \frac{\partial F_1}{\partial X_a} X + F_1(L1_b, X_o) = 0 \tag{4.3}$$

$$\frac{\partial F_2}{\partial L2_a} V_2 + \frac{\partial F_2}{\partial X_a} X + F_2(L2_b, X_o) = 0 \tag{4.4}$$

where V_1 and V_2 are residuals vectors, X_o denotes the approximations to the parameters and X represents the alterations which will be applied to the approximations to obtain the adjusted values X_a. Using the notations:

$$B_1 = \frac{\partial F_1}{\partial L_{1_a}} \qquad B_2 = \frac{\partial F_2}{\partial L_{2_a}}$$

$$A_1 = \frac{\partial F_1}{\partial X_a} \qquad A_2 = \frac{\partial F_2}{\partial X_a}$$

$$W_1 = F_1(L_{1_b}, X_o) \qquad W_2 = F_2(L_{2_b}, X_o)$$

the differential forms (4.3) and (4.4) become:

$$B_1 V_1 + A_1 X + W_1 = 0 \tag{4.5}$$

$$B_2 V_2 + A_2 X + W_2 = 0 \tag{4.6}$$

Since there is only one observable quantity in any equation of the form F_1, B_1 is an identity matrix. Hence (4.5) is reduced to:

$$V_1 + A_1 X + W_1 = 0 \tag{4.7}$$

Further, W_2 for the first cycle of the adjustment will be a null vector because L_{2_b}, the observed values of the ship coordinates, and the approximate values of the ship coordinates, are the same. Also, since there is only one observed quantity, an x or y coordinate of a ship position in any F_2 equation, B_2 is an identity matrix. Therefore (4.6) reduces to:

$$V_2 + A_2 X = 0 \tag{4.8}$$

The function which must be minimized in order to fulfill the principle of least squares requirement is thus, using the Lagrange multipliers method:

$$\Phi = V_1^T P_1 V_1 + V_2^T P_2 V_2 - 2K_1^T(V_1 + A_1 X + W_1) - 2K_2^T(V_2 + A_2 X) \tag{4.9}$$

K_1 and K_2 are column vectors composed of Lagrange multipliers. P_1 is the weight matrix for sextant angle and compass direction observations, and P_2 is the weight matrix for the parameters treated as observations.

Taking the partial derivatives of Φ with respect to V_1, V_2 and X and setting them equal to zero, the three sets of condition equations which must be fulfilled for all observations and parameters are:

$$\frac{1}{2}\frac{\partial\Phi}{\partial V_1} = P_1V_1 - K_1 = 0 \tag{4.10}$$

$$\frac{1}{2}\frac{\partial\Phi}{\partial V_2} = P_2V_2 - K_2 = 0 \tag{4.11}$$

$$\frac{1}{2}\frac{\partial\Phi}{\partial X} = -A_1^TK_1 - A_2^TK_2 = 0 \tag{4.12}$$

The expressions (4.7), (4.8) and (4.10) through (4.12) are a system of five sets of equations having five unknown vectors: V_1, V_2, K_1, K_2 and X. Since the solution of this problem consists of determining the X vector, the five set system is first reduced to a system of two sets of equations having only two unknown vectors.

The procedure for this reduction is begun by first solving (4.11) for V_2 and substituting the resulting expression into (4.8) to give:

$$P_2^{-1}K_2 + A_2X = 0 \tag{4.13}$$

Next, (4.10) is solved for V_1 and the result substituted into (4.7) giving:

$$P_1^{-1}K_1 + A_1X = -W_1 \tag{4.14}$$

Each term in (4.14) is then multiplied by $A_1^TP_1$ to obtain:

$$A_1^TK_1 + A_1^TP_1A_1X = -A_1^TP_1W_1 \tag{4.15}$$

Finally, (4.12) is solved for $A_1^TK_1$ and the result inserted into (4.15) yielding:

$$A_1^TP_1A_1X - A_2^TK_2 = -A_1^TP_1W_1 \tag{4.16}$$

Thus (4.13) and (4.16) are the desired system of two sets of equations with unknown vectors K_2 and X.

In (4.16) the term $A_1^T P_1 A_1$ is the normal equation coefficients array which will be designated N_1. Also, the term $A_1^T P_1 W_1$ is the constant vector of the normal equations, to be designated U. Using these notations, (4.13) and (4.16) become:

$$P_2^{-1} K_2 + A_2 X = 0 \tag{4.17}$$

$$N_1 X - A_2^T K_2 = -U \tag{4.18}$$

Solving (4.17) for K_2,

$$K_2 = -P_2 A_2 X \tag{4.19}$$

which, when substituted into (4.18) gives:

$$N_1 X + A_2^T P_2 A_2 X = -U \tag{4.20}$$

Factoring out X,

$$(N_1 + A_2^T P_2 A_2) X = -U \tag{4.21}$$

But each F_2 equation contains only one unknown parameter, an x or y ship position coordinate. Hence the only non-zero elements of A_2 are - 1's. Further, by virtue of the F_2 structure, A_2 is a square array with non-zero values occurring only in the main diagonal elements corresponding to ship parameters (see Article 4.4.12). Therefore $A_2^T P_2 A_2$ is merely P_2 with non-zero elements only on the main diagonal at positions where dependence is estimated between ship position coordinates as explained in Article 4.4.14. In matrix notation then:

$$A_2^T P_2 A_2 = P_2 \tag{4.22}$$

Substituting (4.22) into (4.21), the factor $(N_1 + P_2)$ in the resultant expression, known as the generalized normals matrix, is denoted as N_{gen}.

Using this notation, (4.21) can be rewritten as:

$$(N_1 + P_2)X = N_{gen}X = -U \tag{4.24}$$

The approximate values X_o of the shore point coordinates and the ship station coordinates are then combined with the alterations determined from (4.24) to obtain the adjusted values of both the terrestrial points and the ship station coordinates.

$$X_a = X_o + X \tag{4.25}$$

If the magnitudes of the final alterations determined by (4.24) indicate that the first approximations X_o are not sufficiently accurate, the adjustment would be recycled. In this situation, the X_a computed from (4.25) for the first cycle would become the X_o for the second cycle of the adjustment. The adjustment would then proceed as outlined above with certain modifications.

The W_2 misclosures vector would no longer be null but would instead consist of the differences between the initially computed adjusted x and y ship coordinates and the originally observed values. With this addition, the W_2 vector would be included in equations (4.8), (4.9) and (4.13). Thus the constant vector U would become:

$$U = A_1^T P_1 W_1 - P_2 W_2 \tag{4.26}$$

With the value of U from (4.26) substituted into (4.23) the problem is solved as previously explained to obtain the new alterations X. In (4.25) the finally adjusted parameters computed from the second cycle would consist of the parameters X plus the approximations X_o which are now the adjusted values computed from the first cycle.

Should an additional recycling be deemed necessary, the adjusted

values from the second cycle would be utilized as the approximations for the third cycle and the operation repeated as described above.

4.4 FORMATION OF EQUATIONS.

The quantities necessary to accomplish the adjustment presented in the previous section are now described, and the expressions which are required in order to perform the indicated computations are developed.

The network parameter notations to be employed are as shown in Figure 4.2. The cartesian coordinates of each ship position S_i are denoted as (x_i, y_i). For each shore point the coordinates correspond

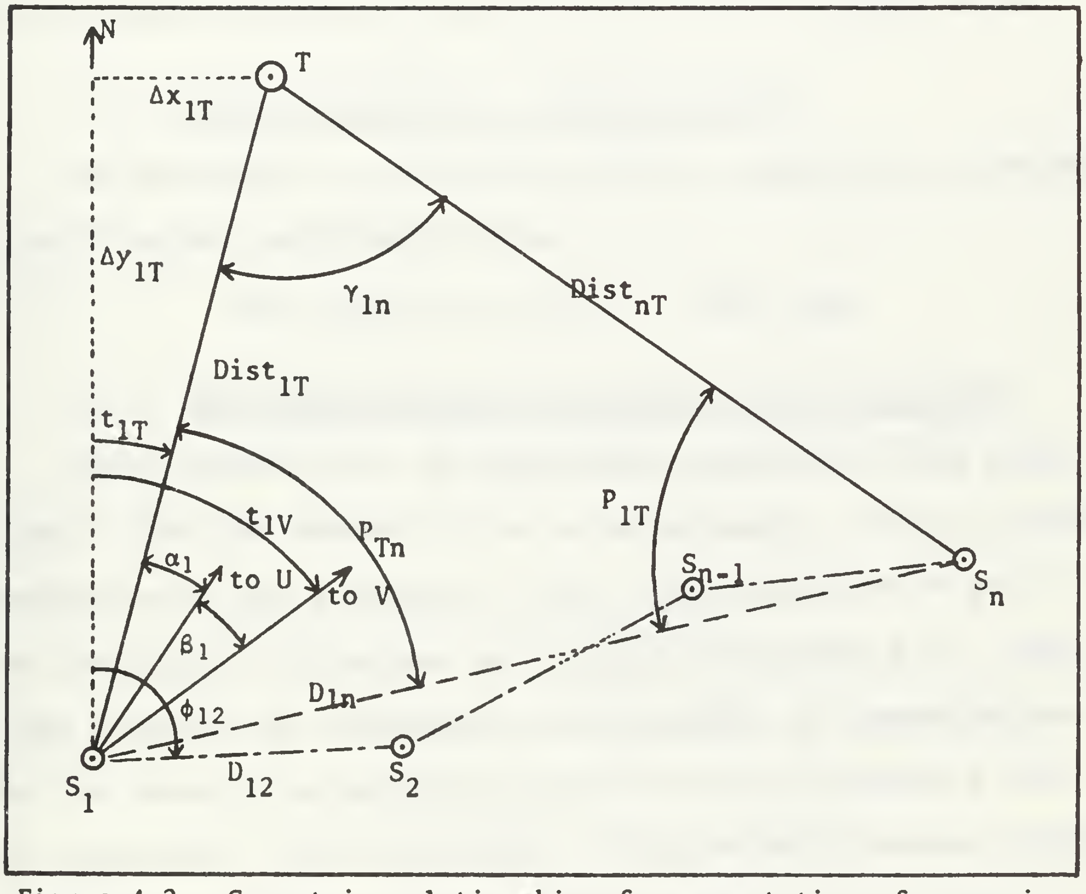

Figure 4.2. Geometric relationships for computation of approximate coordinates of a shore point.

to the point concerned. For example, for terrestrial object V the coordinates are (x_V, y_V). For the purposes of this general development is is assumed that there are three terrestrial points and n ship positions.

4.4.1 Observed Quantities - First Structure.

The quantities L_{1_b} are the directly observed sextant angles

$$\alpha 1_b, \beta 1_b, \alpha 2_b, \beta 2_b, \ldots \alpha n_b, \beta n_b$$

and the directly observed compass bearings

$$t_{1_b}, t_{2_b}, \ldots t_{n_b}$$

to the terrestrial point closest to the direction of ships travel.

4.4.2 Observed Quantities - Second Structure.

The quantities L_{2_b} are the ship position coordinates as determined by an electronic positioning system.

$$(x_{1_b}, y1_b), (x2_b, y2_b), \ldots (x_{n_b}, y_{n_b})$$

4.4.3 Approximate Values of Parameters - First Structure.

These parameters are the approximate coordinates of the shore points. They are computed in the following manner. With the observed coordinates of ship positions S_1 and S_n, the coordinates of point T are determined to illustrate the procedure (See Figure 4.2). Using plane geometric and trigonometric relationships, the approximations for the distances between ship positions and for the angles P and γ are calculated. With these values, the approximate distances from the ship positions to T are computed according to the law of sines. The distance from S_1 to T is thus given by:

$$Dist_{1T} = \frac{\sin P_{1T} \times D_{1n}}{\sin \gamma_{1n}} \tag{4.27}$$

With the observed compass bearing t_{1V} to terrestrial point V and the observed sextant angles α_1 and β_1, the approximate azimuth t_{1T} is computed.

$$t_{1T} = t_{1V} - (\alpha_1 + \beta_1) \tag{4.28}$$

The horizontal and vertical increments Δx_{1T} and Δy_{1T} are then written as:

$$\begin{aligned} \Delta x_{1T} &= Dist_{1T} \times \sin t_{1T} \\ \Delta y_{1T} &= Dist_{1T} \times \cos t_{1T} \end{aligned} \tag{4.29}$$

The approximate coordinates of point T are thus expressed as:

$$\begin{aligned} x_{T_o} &= x_{1_o} + \Delta x_{1T} \\ y_{T_o} &= y_{1_o} + \Delta y_{1T} \end{aligned} \tag{4.30}$$

In similar manner the approximate coordinates of points U and V are determined.

4.4.4 Approximate Values of Parameters - Second Structure.

For ship coordinates, these parameters are the observed quantities L_{2_b} described in Article 4.4.2.

4.4.5 Computed Values of Sextant Angles.

The general expression for a computed sextant angle formed between shore points J and K, subtended at ship position S_i is given by:

$$\alpha_i = \tan^{-1}\left[\frac{x_J - x_i}{y_J - y_i}\right] - \tan^{-1}\left[\frac{x_K - x_i}{y_K - y_i}\right] \tag{4.31}$$

4.4.6 <u>Computed Values of Compass Bearings.</u>

The general expression for a computed compass bearing from ship position S_i to shore point J is:

$$t_i = \tan^{-1}\left[\frac{x_J - x_i}{y_J - y_i}\right] \tag{4.32}$$

4.4.7 <u>First Mathematical Structure.</u>

This is the model of the form $F_1(L_{1_a}, X_a) = 0$. An expression is developed for each sextant angle and compass bearing.

$$\begin{aligned}
\tan^{-1}\left[\frac{x_{U_a} - x_{1_a}}{y_{V_a} \quad y_{1_a}}\right] - \tan^{-1}\left[\frac{x_{T_a} - x_{1_a}}{y_{T_a} \quad y_{1_a}}\right] - \alpha_{1_a} &= 0\\
\tan^{-1}\left[\frac{x_{V_a} - x_{1_a}}{y_{V_a} - y_{1_a}}\right] - \tan^{-1}\left[\frac{x_{U_a} - x_{1_a}}{y_{U_a} - y_{1_a}}\right] - \beta_{1_a} &= 0\\
\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \quad &\ \ \vdots\\
\tan^{-1}\left[\frac{x_{U_a} - x_{n_a}}{y_{U_a} - y_{n_a}}\right] - \tan^{-1}\left[\frac{x_{T_a} - x_{n_a}}{y_{T_a} - y_{n_a}}\right] - \alpha_{n_a} &= 0\\
\tan^{-1}\left[\frac{x_{V_a} - x_{n_a}}{y_{V_a} - y_{n_a}}\right] - \tan^{-1}\left[\frac{x_{U_a} - x_{n_a}}{y_{U_a} - y_{n_a}}\right] - \beta_{n_a} &= 0\\
\tan^{-1}\left[\frac{x_{V_a} - x_{1_a}}{y_{V_a} - y_{1_a}}\right] - t_{1_a} &= 0\\
\vdots \qquad\qquad\qquad \vdots \quad &\ \ \vdots\\
\tan^{-1}\left[\frac{x_{V_a} - x_{n_a}}{y_{V_a} - y_{n_a}}\right] - t_{n_a} &= 0
\end{aligned} \tag{4.33}$$

4.4.8 <u>Second Mathematical Structure.</u>

This model, the parameters treated as observations, is expressed as $F_2(L_{2_a}, X_a) = 0$. The shore point parameters are only treated as

observations in the sense of being observations with zero weight. Thus shore coordinate functions do not enter this structure. To present the ship parameter portion of the structure in a more meaningful manner, the relationships $L_{2_a} = L_{2_b} + V_2$ and (4.25) are substituted permitting the observation equations to be written. They take the following form:

$$
\begin{aligned}
x_{1_b} &+ v_{2_1} - \Delta x_1 = 0 \\
y_{1_b} &+ v_{2_2} - \Delta y_1 = 0 \\
x_{2_b} &+ v_{2_3} - \Delta x_2 = 0 \\
y_{2_b} &+ v_{2_4} - \Delta y_2 = 0 \\
&\vdots \\
x_{n_b} &+ v_{2_{2n-1}} - \Delta x_n = 0 \\
y_{n_b} &+ v_{2_{2n}} - \Delta y_n = 0
\end{aligned}
\tag{4.34}
$$

4.4.9 <u>Misclosure Vector - First Structure.</u>

The misclosures W_1 are obtained by evaluating the mathematical model F_1 with L_{1_b} and X_o. To illustrate the procedure, the expressions for the first sextant angle and the first compass direction are shown below. Since there are three observations at each of the n ship positions, W_1 consists of a total of 3n elements.

$$\rho' \tan^{-1}\left[\frac{x_{U_o} - x_{1_o}}{y_{U_o} - y_{1_o}}\right] - \rho' \tan^{-1}\left[\frac{x_{T_o} - x_{1_o}}{y_{T_o} - y_{1_o}}\right] - \alpha_{1_b}' = w_{1_1} \tag{4.35}$$

$$\rho' \tan^{-1}\left[\frac{x_{V_o} - x_{1_o}}{y_{V_o} - y_{1_o}}\right] - t_{1_b}' = w_{1_{2n+1}} \tag{4.36}$$

4.4.10 Misclosure Vector - Second Structure.

As explained in Section 4.3, the W_2 matrix is null for the first cycle since L_{2_b} and X_o are the same quantities. However, for the recycled adjustment for ship parameters, (x_{i_o}, y_{i_o}) are the adjusted coordinates computed from the first cycle and (x_{i_b}, y_{i_b}) are the observed values from the electronic positioning system. In this case, that portion of W_2 corresponding to ship coordinates takes the following form:

$$
\begin{aligned}
x_{1_o} - x_{1_b} &= w_{2_1} \\
y_{1_o} - y_{1_b} &= w_{2_2} \\
x_{2_o} - x_{2_b} &= w_{2_3} \\
y_{2_o} - y_{2_b} &= w_{2_4} \\
\vdots \quad & \quad \vdots \\
x_{n_o} - x_{n_b} &= w_{2_{2n-1}} \\
y_{n_o} - y_{n_b} &= w_{2_{2n}}
\end{aligned}
\tag{4.37}
$$

4.4.11 Direction Coefficients - First Structure.

This array, of the general form $A_1 = \partial F_1/\partial X_a$, is obtained by taking the partial derivatives of the structure expressions with respect to each of the unknown parameters. For ease in computation and presentation, the A_1 array is partitioned into two parts, $\overset{o}{A}$ representing the partials with respect to the terrestrial object parameters and $\bar{A}$ designating the partials with respect to ship position parameters.

$$
A_1 = \left[\overset{o}{A} \;\vdots\; \bar{A} \right] \tag{4.38}
$$

Developing first the $\overset{o}{A}$ array, the partial derivatives for α_1 and t_1 are presented to illustrate the types of expressions which make up the elements of the matrix.

$$
\begin{aligned}
\frac{\partial \alpha_1}{\partial x_T} &= \frac{-\rho'(y_T - y_1)}{(x_T - x_1)^2 + (y_T - y_1)^2} \\
\frac{\partial \alpha_1}{\partial y_T} &= \frac{+\rho'(x_T - x_1)}{(x_T - x_1)^2 + (y_T - y_1)^2} \\
\frac{\partial \alpha_1}{\partial x_U} &= \frac{+\rho'(y_U - y_1)}{(x_U - x_1)^2 + (y_U - y_1)^2} \\
\frac{\partial \alpha_1}{\partial y_U} &= \frac{-\rho'(x_U - x_1)}{(x_U - x_1)^2 + (y_U - y_1)^2} \\
\frac{\partial t_1}{\partial x_V} &= \frac{-\rho'(y_V - y_1)}{(x_V - x_1)^2 + (y_V - y_1)^2} \\
\frac{\partial t_1}{\partial y_V} &= \frac{+\rho'(x_V - x_1)}{(x_V - x_1)^2 + (y_V - y_1)^2}
\end{aligned}
\qquad (4.39)
$$

Since partials are taken with respect to the x and y coordinates of each of three shore points, the $\overset{o}{A}$ matrix has a column dimension of six. Because there are three observations, two sextant angles and a compass bearing, at each of the n positions of the ship, the $\overset{o}{A}$ matrix has a row dimension of 3n. The arrangement of the partial derivative elements is shown in the schematic array on page 64.

$$
{}_{3n}\overset{o}{A}_{6} = \begin{bmatrix}
\frac{\partial\alpha_1}{\partial x_T} & \frac{\partial\alpha_1}{\partial y_T} & \frac{\partial\alpha_1}{\partial x_U} & \frac{\partial\alpha_1}{\partial y_U} & 0 & 0 \\
0 & 0 & \frac{\partial\beta_1}{\partial x_U} & \frac{\partial\beta_1}{\partial y_U} & \frac{\partial\beta_1}{\partial x_V} & \frac{\partial\beta_1}{\partial y_V} \\
\frac{\partial\alpha_2}{\partial x_T} & \frac{\partial\alpha_2}{\partial y_T} & \frac{\partial\alpha_2}{\partial x_U} & \frac{\partial\alpha_2}{\partial y_U} & 0 & 0 \\
0 & 0 & \frac{\partial\beta_2}{\partial x_U} & \frac{\partial\beta_2}{\partial y_U} & \frac{\partial\beta_2}{\partial x_V} & \frac{\partial\beta_2}{\partial y_V} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial\alpha_n}{\partial x_T} & \frac{\partial\alpha_n}{\partial y_T} & \frac{\partial\alpha_n}{\partial x_U} & \frac{\partial\alpha_n}{\partial y_U} & 0 & 0 \\
0 & 0 & \frac{\partial\beta_n}{\partial x_U} & \frac{\partial\beta_n}{\partial y_U} & \frac{\partial\beta_n}{\partial x_V} & \frac{\partial\beta_n}{\partial y_V} \\
0 & 0 & 0 & 0 & \frac{\partial t_1}{\partial x_V} & \frac{\partial t_1}{\partial y_V} \\
0 & 0 & 0 & 0 & \frac{\partial t_2}{\partial x_V} & \frac{\partial t_2}{\partial y_V} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \frac{\partial t_n}{\partial x_V} & \frac{\partial t_n}{\partial y_V}
\end{bmatrix} \tag{4.40}
$$

Next, the $\bar{A}$ array is developed. Again the partial derivatives for α_1 and t_1 are presented to illustrate the expressions of which the matrix is composed.

$$
\begin{aligned}
\frac{\partial\alpha_1}{\partial x_1} &= -\frac{\rho'(y_U - y_1)}{(x_U - x_1)^2 + (y_U - y_1)^2} + \frac{\rho'(y_T - y_1)}{(x_T - x_1)^2 + (y_T - y_1)^2} \\
\frac{\partial\alpha_1}{\partial y_1} &= +\frac{\rho'(x_U - x_1)}{(x_U - x_1)^2 + (y_U - y_1)^2} - \frac{\rho'(x_T - x_1)}{(x_T - x_1)^2 + (y_T - y_1)^2}
\end{aligned} \tag{4.41}
$$

$$\frac{\partial t_1}{\partial x_1} = - \frac{\rho'(y_V - y_1)}{(x_V - x_1)^2 + (y_V - y_1)^2}$$
$$\frac{\partial t_1}{\partial y_1} = + \frac{\rho'(x_V - x_1)}{(x_V - x_1)^2 + (y_V - y_1)^2} \qquad (4.41)$$

Because partials are taken with respect to the x and y coordinates of each of the n ship positions, the $\bar{A}$ matrix has 2n columns. Since there are three observations at each ship position, the row dimension is 3n. The matrix array of the partial derivative elements is:

$$
{}_{3n}\bar{A}_{2n} = \begin{bmatrix}
\frac{\partial\alpha_1}{\partial x_1} & \frac{\partial\alpha_1}{\partial y_1} & 0 & 0 & \cdots & 0 & 0 \\
\frac{\partial\beta_1}{\partial x_1} & \frac{\partial\beta_1}{\partial y_1} & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \frac{\partial\alpha_2}{\partial x_2} & \frac{\partial\alpha_2}{\partial y_2} & \cdots & 0 & 0 \\
0 & 0 & \frac{\partial\beta_2}{\partial x_2} & \frac{\partial\beta_2}{\partial y_2} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \frac{\partial\alpha_n}{\partial x_n} & \frac{\partial\alpha_n}{\partial y_n} \\
0 & 0 & 0 & 0 & \cdots & \frac{\partial\beta_n}{\partial x_n} & \frac{\partial\beta_n}{\partial y_n} \\
\frac{\partial t_1}{\partial x_1} & \frac{\partial t_1}{\partial y_1} & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & \frac{\partial t_2}{\partial x_2} & \frac{\partial t_2}{\partial y_2} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \frac{\partial t_n}{\partial x_n} & \frac{\partial t_n}{\partial y_n}
\end{bmatrix} \qquad (4.42)
$$

4.4.12 Direction Coefficients - Second Structure.

This array, of the general form $A_2 = \partial F_2/\partial X_a$, is obtained by taking the partial derivatives of the structure expressions with respect to each of the parameters. Since the shore point coordinate parameters are treated as observations with zero weight, the partial derivatives for these parameters are simply zero. For ship coordinates, the partial derivatives are:

$$
\begin{aligned}
\frac{\partial f_1}{\partial x_1} &= \frac{\partial(-x_1)}{\partial x_1} = -1 \\
\frac{\partial f_2}{\partial y_1} &= \frac{\partial(-y_1)}{\partial y_1} = -1 \\
\frac{\partial f_3}{\partial x_2} &= \frac{\partial(-x_2)}{\partial x_2} = -1 \\
\frac{\partial f_4}{\partial y_2} &= \frac{\partial(-y_2)}{\partial y_2} = -1 \\
&\vdots \\
\frac{\partial f_{2n-1}}{\partial x_n} &= \frac{\partial(-x_n)}{\partial x_n} = -1 \\
\frac{\partial f_{2n}}{\partial y_n} &= \frac{\partial(-y_n)}{\partial y_n} = -1
\end{aligned}
\tag{4.43}
$$

Since the partial derivatives are taken with respect to the x and y coordinates of each of three shore points and n ship positions, the A_2 matrix has a column dimension of 2n+6. Because there are two observations, an x and y coordinate, at each of the three shore points and n ship positions, the A_2 matrix also has a row dimension of 2n+6. Further, it is a diagonal array with the only non-zero elements (-1's) occurring in positions corresponding to ship coordinates.

4.4.13 Weight Matrix - First Structure.

For sextant angles, it is assumed that the standard errors for all observations are equal. The same assumption is also made for compass directions. That is,

$$
\begin{aligned}
m_{\alpha_1} = m_{\beta_1} = m_{\alpha_2} = m_{\beta_2} \ . \ . \ . = m_{\alpha} \\
m_{t_1} = m_{t_2} . \ . \ . = m_t
\end{aligned}
\qquad (4.44)
$$

Since all observations are considered to be independent, P_1 is a diagonal matrix of the reciprocals of the sextant angle and compass bearing standard errors squared. Since there are three observations at each ship position, the square dimension of P_1 is 3n.

4.4.15 Weight Matrix - Second Structure.

P_2 is the weight matrix for all observed parameters. Since the shore coordinates are treated as completely unknown parameters (observations with zero weight), P_2 takes the general matrix form:

$$
{}_{2n+6}P_{2\,2n+6} = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \Sigma_{\hat{x}}^{-1} \end{array}\right] \qquad (4.45)
$$

$\Sigma_{\hat{x}}$ is the variance-covariance matrix associated with the ship coordinates. In its most generalized form, it would be entirely filled with variance and covariance estimates. If, however, it is assumed that systematic errors which would affect all x and y ship coordinates can be ascertained and removed or that the positioning method is being utilized as a relative "localized" system, the successive x coordinates and successive y coordinates can be considered as independently determined quantities. The array would then consist of "n" 2 × 2 blocks on

the diagonal, each block representing a ship position. The four elements of each block are the estimated variances of the x and y coordinates on the principal diagonal and the covariance between x and y in the off diagonal positions. If for each ship position the x and y coordinate determinations are considered as independent quantities, then $\Sigma_{\hat{x}}$ becomes a diagonal matrix. Further, if all coordinate values are assumed to be of equal accuracy, $\Sigma_{\hat{x}}$ is a unit matrix. For the localized system 2 × 2 block form which is utilized in this problem, $\Sigma_{\hat{x}}$ is represented as follows:

$$
{}_{2n}\Sigma_{\hat{x}\,2n} = \begin{bmatrix}
m^2_{x_1} & m_{x_1y_1} & 0 & 0 & \cdots & 0 & 0 \\
m_{x_1y_1} & m^2_{y_1} & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & m^2_{x_2} & m_{x_2y_2} & \cdots & 0 & 0 \\
0 & 0 & m_{x_2y_2} & m^2_{y_2} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & m^2_{x_n} & m_{x_ny_n} \\
0 & 0 & 0 & 0 & \cdots & m_{x_ny_n} & m^2_{y_n}
\end{bmatrix} \tag{4.46}
$$

4.4.15 Normals Matrix - First Structure.

-th the A_1 matrix partitioned into $\overset{o}{A}$ and $\bar{A}$, the normals matrix N_1 is formed as follows:

$$
N_1 = A_1^T P_1 A_1 = \begin{bmatrix} \overset{o}{A}{}^T \\ \hline \bar{A}^T \end{bmatrix} \begin{bmatrix} P_1 \end{bmatrix} \left[\overset{o}{A} \;\vdots\; \bar{A} \right] = \begin{bmatrix} \overset{o}{A}{}^T P_1 \overset{o}{A} & \overset{o}{A}{}^T P_1 \bar{A} \\ \hline \bar{A}^T P_1 \overset{o}{A} & \bar{A}^T P_1 \bar{A} \end{bmatrix} \tag{4.47}
$$

Now, setting $\overset{o}{A}{}^T P_1 \bar{A}$ equal to $\tilde{N}$, the expression for N_1 may be written as:

$$N_1 = \left[\begin{array}{c:c} \overset{o}{N} & \tilde{N} \\ \hdashline \tilde{N}^T & \bar{N} \end{array}\right] \tag{4.48}$$

4.4.16 Generalized Normals Matrix.

The generalized normals matrix N_{gen} is formed by adding P_2 to N_1. The square dimension of N_{gen} is 2n+6.

$$N_{gen} = \left[\begin{array}{c:c} \overset{o}{N} & \tilde{N} \\ \hdashline \tilde{N}^T & \bar{N} + \Sigma_{\hat{x}}^{-1} \end{array}\right] \tag{4.49}$$

4.4.17 Constant Vector of Normal Equations.

The vector U is formed as explained in Section 4.3 wherein the component matrices are as described in previous articles of this section. For both the first and recycled adjustment, U consists of 2n+6 elements representing the x and y coordinates of each shore point and ship position. For the first cycle, U is formed as follows:

$$U = A_1^T P_1 W_1 = \left[\begin{array}{c} \overset{o}{A}{}^T \\ \hdashline \bar{A}^T \end{array}\right] \left[P_1\right] \left[W_1\right] = \left[\begin{array}{c} \overset{o}{A}{}^T P_1 W_1 \\ \hdashline \bar{A}^T P_1 W_1 \end{array}\right] \tag{4.50}$$

When the adjustment is recycled, U becomes:

$$U = A_1^T P_1 W_1 - P_2 W_2 = \left[\begin{array}{c} \overset{o}{A}{}^T P_1 W_1 \\ \hdashline \bar{A}^T P_1 W_1 \end{array}\right] - \left[P_2\right] \left[W_2\right] \tag{4.51}$$

4.4.18 Adjusted Parameters.

The corrections X obtained by solving (4.24) are combined with the approximate values of the shore points and observed ship positions as shown in (4.25) to obtain the finally adjusted values X_a of the

terrestrial objects and ship position coordinates. Since there are n ship positions and three shore points, each with an x and y coordinate, the X_a vector is composed of 2n+6 elements.

$$X_a = \begin{bmatrix} x_{T_a} \\ y_{T_a} \\ x_{U_a} \\ y_{U_a} \\ x_{V_a} \\ y_{V_a} \\ x_{1_a} \\ y_{1_a} \\ \vdots \\ x_{n_a} \\ y_{n_a} \end{bmatrix} = \begin{bmatrix} x_{T_o} + \Delta x_T \\ y_{T_o} + \Delta y_T \\ x_{U_o} + \Delta x_U \\ y_{U_o} + \Delta y_U \\ x_{V_o} + \Delta x_V \\ y_{V_o} + \Delta y_V \\ x_{1_o} + \Delta x_1 \\ y_{1_o} + \Delta y_1 \\ \vdots \\ x_{n_o} + \Delta x_n \\ y_{n_o} + \Delta y_n \end{bmatrix} \qquad (4.52)$$

4.4.19 <u>Adjusted Sextant Angles and Compass Bearings.</u>

These values, $L1_a$, are obtained by adding the residuals vector V_1 computed using (4.7) to the originally observed sextant angles and compass directions $L1_b$. Since there are three such observations at each ship position, $L1_a$ consists of 3n elements.

$$L_{1_a} = \begin{bmatrix} l_{1_a} \\ l_{2_a} \\ l_{3_a} \\ \cdot \\ \cdot \\ \cdot \\ l_{3n-1_a} \\ l_{3n_a} \end{bmatrix} = \begin{bmatrix} l_{1_b} & + v_1 \\ l_{2_b} & + v_2 \\ l_{3_b} & + v_3 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ l_{3n-1_b} & + v_{3n-1} \\ l_{3n_b} & + v_{3n} \end{bmatrix} \tag{4.53}$$

4.5 ERROR ANALYSIS.

In order to examine the accuracies of the adjusted values of the parameters and the observed quantities, the variance-covariance matrices for each set of values is obtained. The sequence of operations necessary to arrive at these arrays is explained below, continuing the matrix notations utilized in the previous two sections.

4.5.1 Variance of Unit Weight.

The variance of unit weight, denoted as m_o^2, is first determined. All weighting in the problem is based on the same variance of unit weight. This unit weight variance is expressed as the sum of squares of the weighted residuals divided by the degrees of freedom.

$$m_o^2 = \frac{V^T PV}{\text{degrees of freedom}} \tag{4.54}$$

The weighted residuals square sum for the entire problem is the sum of the square sums from each of the two mathematical structures.

$$V^T PV = V_1^T P_1 V_1 + V_2^T P_2 V_2 \tag{4.55}$$

In partitioned matrix form, (4.55)is expressed as:

$$V^TPV = \left[V_1^T \;\vdots\; V_2^T \right] \left[\begin{array}{c|c} P_1 & 0 \\ \hline 0 & P_2 \end{array} \right] \left[\begin{array}{c} V_1 \\ \hline V_2 \end{array} \right] \tag{4.56}$$

P_1 is explained in Article 4.4.13 and P_2 in 4.4.14. The square dimensions of P_1 and P_2 are 3n and 2n+6 respectively. V_1 is computed from (4.7) and is a column vector of 3n elements. V_2 is calculated using (4.8) and consists of 2n+6 elements. The last 2n elements are simply the negative X vector of alterations to ship coordinates.

The number of degrees of freedom is determined as follows:

$$\text{degrees of freedom} = r + s - u \tag{4.57}$$

where

r = the number of sextant angle and compass observations.

s = the number of ship position x and y coordinates.

u = the number of shore point x and y coordinates.

For the general problem with three shore points and n ship positions,

$$r = 3n, \qquad s = 2n \qquad \text{and} \qquad u = 6 \tag{4.58}$$

Hence the number of degrees of freedom, substituting (4.58) into (4.57), is 5n - 6. Thus the variance of unit weight is finally expressed as:

$$m_o^2 = \frac{V_1^TP_1V_1 + V_2^TP_2V_2}{5n - 6} \tag{4.59}$$

4.5.2 <u>Variance-Covariance Matrix of Parameters.</u>

The weignt coefficient matrix of the unknown parameters is simply the inverse of the generalized normals matrix developed in Article 4.4.16.

$$Q_X = N^{-1}_{gen} \tag{4.60}$$

Thus the variance-covariance matrix Σ_X which indicates the accuracy of the adjusted values of the parameters is:

$$\Sigma_X = m_o^2 \, Q_X \tag{4.61}$$

It is a square array of dimension 2n+6.

4.5.3 Variance-Covariance Matrix of Observations.

The weight coefficient matrix for the adjusted values of the observed sextant angles and compass directions is:

$$Q_{L_{1_a}} = A_1 N_1^{-1} A_1^T \tag{4.62}$$

where A_1 and N_1 are as described in Articles 4.4.11 and 4.4.15 respectively. Its square dimension is 3n since two sextant angles and a compass direction are observed at each ship position. Multiplying (4.62) by the variance of unit weight from (4.59):

$$\Sigma_{L_{1_a}} = m_o^2 \, A_1 N_1^{-1} A_1^T \tag{4.63}$$

4.6 NUMERICAL EXAMPLE.

The unavailability of actual hydrographic survey data in a form which would lend itself to adaptation to this problem necessitated the compilation of fictitious information with which to illustrate the computation procedure. In order to simulate the actual observation situation as closely as possible, the following method was employed. A 28" × 30" plotting sheet prepared by the Naval Oceanographic Office was used as the basis for generating the required information. The plotting sheet contained a circular lattice net, longitude and latitude

and a UTM X-Y grid coordinate system. Since standard computer program routines are available for converting coordinates from any of these three systems to the other two, the X-Y grid was utilized for the purposes of this example. In a practical situation, the electronic positions would be recorded and later converted with a computer program to the desired X and Y coordinate values.

The computational procedure for this problem was programmed in the Fortran IV language and run on the IBM 7094 computer at the Numerical Computation Laboratory of The Ohio State University. Free electronic computer services were made available by the University for all programming carried out in this investigation.

Considerable effort was expended in designing the program for this problem in as general a form as possible to allow for variations in the types and amount of input information. Many combinations of ship headings and observation data were tested to insure that the program would produce proper results for all conceivable situations. The numerical example presented in this section is accomplished with three shore points and four ship positions. However, the number of each type of position can be increased as desired depending upon the amount of input data to be processed.

4.6.1 Problem Input Information.

(a) Coordinates for each of four ship positions.

Source: arbitrarily placed on plotting sheet.

Data units: meters.

Ship Position	X	Y
1	299865.0	1960800.0
2	299992.0	1961573.0
3	300081.0	1962378.0
4	300224.5	1963203.0

(b) Sextant angles and compass directions observed at each ship position.

Source: Measurements made on plotting sheet with three-arm protractor.

Ship Position	α	β	t
1	31° 33'	24° 25'	$325\overset{\circ}{.}0$
2	32° 04'	29° 00'	$316\overset{\circ}{.}5$
3	29° 37'	32° 36'	$305\overset{\circ}{.}7$
4	24° 35'	32° 20'	$291\overset{\circ}{.}2$

(c) Side of survey ship from which angle and direction observations are made: Port side.

(d) Variance and covariance estimates for electronic positioning system coordinates of ship positions.

Source: Arbitrarily selected, based upon information obtained from various publications describing accuracies achievable with short-range electronic positioning systems.

Data units: meters squared.

Ship Position	X variance	Y variance	Covariance
1	15.1	18.6	+2.1
2	20.8	19.4	+2.4
3	17.9	19.9	+1.8
4	16.1	20.8	+2.0

(e) Standard errors of sextant angle and gyro compass observations.

Sources: Sextant angle - from estimates given in JEFFERS(7) and FAGERHOLM and THUNBERG(21). Gyro compass bearing - arbitrarily selected based on discussion presented in BOWDITCH(3).

Sextant angle observation standard error: $1'.0$

Gyro compass observation standard error: $5'.0$

4.6.2 Auxiliary Information Computed by Program.

(a) The azimuth from each ship position to each succeeding position.

ϕ_{12}	$009^\circ.330$	ϕ_{14}	$008^\circ.509$	ϕ_{24}	$008^\circ.118$
ϕ_{13}	$007^\circ.794$	ϕ_{23}	$006^\circ.309$	ϕ_{34}	$009^\circ.867$

(b) Approximate angles γ subtended at each shore position between each set of ship positions. A partial listing, for the angles between ship positions 1 and 4, is as follows:

At point T	At point U	At point V
$34^\circ.750$	$41^\circ.717$	$33^\circ.800$

(c) Approximate coordinates of each shore point (in meters):

Point	X	Y
T	296810.8	1960748.4
U	296904.6	1962549.6
V	297421.0	1964290.4

4.6.3 Adjusted Values and Associated Standard Errors.

(a) Ship positions (in meters):

Position	X	m	Y	m
1	299864.8	1.6	1960798.2	1.6
2	299990.8	1.2	1961574.4	1.4
3	300082.2	1.3	1962380.7	1.3
4	300224.8	1.5	1963201.7	1.6

(b) Shore points (in meters):

Point	X	m	Y	m
T	296810.2	7.0	1960747.8	2.3
U	296902.1	4.1	1962549.7	1.9
V	297416.6	4.1	1964292.4	3.2

(c) Sextant angles:

Position	α	m	β	m
1	$31^\circ\ 33'.0$	$1'.0$	$24^\circ\ 25'.2$	$0'.7$
2	$32^\circ\ 03'.7$	$0'.9$	$29^\circ\ 00'.0$	$0'.8$
3	$29^\circ\ 37'.7$	$1'.0$	$32^\circ\ 35'.7$	$1'.0$
4	$24^\circ\ 34'.6$	$1'.1$	$32^\circ\ 20'.2$	$0'.9$

(d) Compass directions:

Position	t	m
1	$325^\circ.02$	$5'.0$
2	$316^\circ.44$	$5'.0$
3	$305^\circ.75$	$5'.0$
4	$291^\circ.17$	$5'.0$

5. DETERMINATION OF SHIP POSITIONS FROM VISUAL MEASUREMENTS AT SHORE CONTROL POINTS

5.1 GENERAL.

This problem consists of locating the ship position by directions observed from the shore, as described in Article 2.3.1. For the purposes of the adjustment, it is assumed that a series of ship positions are to be determined in order to either calibrate an electronic positioning system or to determine precise sounding locations for a nautical charting or engineering project. The ship will be observed from a number of shore stations at specified time intervals as it proceeds slowly along the coast in the survey area.

In practice, directions are usually observed to the ship with a continuous tracking azimuth instrument which permits readings to one-hundredth of a degree to be made. The instrument is initially oriented on another known station. The angle to the ship is then measured to obtain the ship direction. This type of instrument is advantageous for the situation where the ship is moving at a fairly rapid rate of speed as the reading is being taken. It does not, however, permit the maximum possible direction accuracy to be achieved. In order that this be accomplished, a theodolite capable of considerably greater precision would be employed. For the purposes of this problem, where maximum accuracy is desired, it is assumed that the theodolite is used with the vessel either anchored or moving slowly enough to permit satisfactory directions to be observed to it.

The ship positions to be determined may be considered as being independent on one another since no connections are made between them. Thus each one can be determined by a separate adjustment, utilizing in each case all of the information observed at the shore points which relates to the ship position concerned.

5.2 PROBLEM OBSERVATION DATA.

Referring to Figure 5.1, at each occupied shore station (T, U, V and W) two directions are observed to known network stations (1, 2, 3,

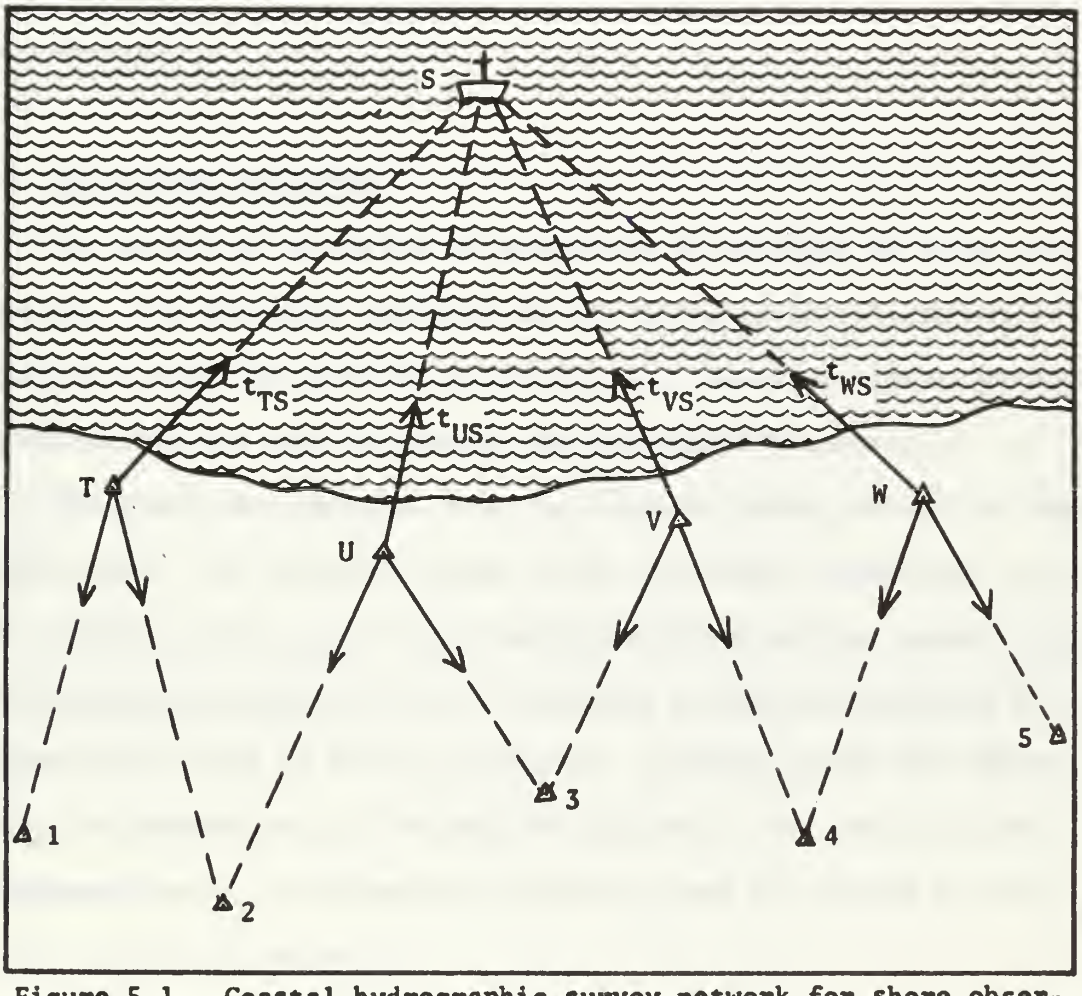

Figure 5.1. Coastal hydrographic survey network for shore observations.

4 and 5). The directions observed to the known stations, together with azimuths computed using the coordinates of the known points, are employed to calculate orienting angles for the later determination of the orientation of intersecting azimuths to the ship.

The second part of the problem consists of azimuths being simultaneously observed to the ship target S from the occupied shore stations T, U, V and W at time intervals dependent on the ship's speed and the survey requirements. Each time a set of directions is observed to the ship, the electronic positioning system coordinates of the ship are also recorded, as well as depth information and any other hydrographic data desired.

5.3 ADJUSTMENT PROCEDURE.

The method of variation of parameters is utilized to solve this problem. The procedure employed for the intersection adjustment is based on a solution given by RICHARDUS(22). Matrix notations according to UOTILA(20) are used to present the adjustment procedure.

The directions observed from the occupied shore stations to the ship compose the observed values of the observable quantities, L_b. The adjusted values of the observable quantities are designated as L_a. The unknown parameters are the coordinates of the ship position S. The approximate values of these coordinates, computed before the adjustment, are denoted as X_o. The values obtained as the result of the adjustment are X_a. The amount by which X_o must be altered to give X_a is the correction vector X.

$$X_a = X_o + X \tag{5.1}$$

The mathematical structure which is selected for the adjustment is a system of equations expressing the adjusted observed quantities as functions of the adjusted parameters.

$$L_a = F(X_a) \tag{5.2}$$

Using (5.2) with the computed approximations X_o of the unknown parameters, the numerical values of the observed quantities can be rigorously computed through the mathematical structure.

$$L_o = F(X_o) \tag{5.3}$$

Now the difference between L_o from (5.3) and the observed values L_b is the corrections vector L.

$$L = L_o - L_b \tag{5.4}$$

The residuals vector of observations, V, is expressed as the difference between the adjusted and observed observations defined above.

$$V = L_a - L_b \tag{5.5}$$

But it has been shown that the adjusted observations are a function of the adjusted unknown parameters (5.2). Therefore, the residuals (5.5) can be written as:

$$V = F(X_a) - L_b \tag{5.6}$$

Now it was also explained that the adjusted unknowns are the approximate parameters plus corrections (5.1). Hence (5.6) can be expressed as:

$$L_b + V = F(X_o + X) \tag{5.7}$$

Linearizing using the Taylor series development:

$$L_b + V = F(X_o) + AX \tag{5.8}$$

where A is the direction coefficients matrix, the partial derivatives

of L with respect to X.

Substituting (5.3) into (5.8) and using (5.4), the result is simplified to:

$$V = AX + L \tag{5.9}$$

which is the expression for the observation equations.

The function which must be minimized in order to fulfill the principle of least squares requirement is now:

$$\Phi = V^T PV \tag{5.10}$$

where P is the weight matrix of the observed quantities. Substituting (5.9) into (5.10):

$$\Phi = (AX + L)^T P(AX + L) \tag{5.11}$$

Expanding the right side:

$$\Phi = X^T A^T PAX + X^T A^T PL + L^T PAX + L^T PL \tag{5.12}$$

Now since X and L are vectors and P is symmetric ($P = P^T$):

$$X^T A^T PL = L^T PAX \tag{5.13}$$

Thus (5.12) can be written as:

$$\Phi = X^T A^T PAX + 2X^T A^T PL + L^T PL \tag{5.14}$$

Designating the normals matrix $A^T PA$ as N and the constant vector $A^T PL$ as U, (5.14) takes the form:

$$\Phi = X^T NX + 2X^T U + L^T PL \tag{5.15}$$

Taking the partial derivatives of Φ with respect to X and setting them equal to zero, the condition which must be fulfilled for $V^T PV$ to be a minimum is:

$$\frac{1}{2}\frac{\partial\Phi}{\partial X^T} = X^T N^T + U^T = 0 \tag{5.16}$$

or

$$NX + U = 0 \tag{5.17}$$

By solving (5.17) for X, the corrections to the approximate values of the unknown parameters are:

$$X = -N^{-1}U \tag{5.18}$$

Substituting these corrections into (5.1), the desired adjusted values of the unknown parameters, the coordinates of the ship position, are obtained.

The adjusted directions to the ship are obtained by solving (5.9) for V and substituting the result into (5.5) to permit the computation of L_a.

5.4 FORMATION OF EQUATIONS.

The quantities necessary to accomplish the adjustment procedure presented in Section 5.3 are now described and expressions required are developed. The notations used are as indicated in Figures 5.2 and 5.3.

5.4.1 Approximate Values of Unknown Parameters.

The method employed for the computation of approximate ship coordinates is similar to the procedure described in Article 4.4.3 except that the terrestrial and ship information is interchanged.

Using plane geometric and trigonometric relationships, approximations for the angles labeled Q and γ in Figure 5.2 are first determined. With the computed distances D between shore positions, the approximate distances from shore stations to the ship are computed by utilizing the law of sines. For example, from shore point T to the ship S:

$$\mathrm{Dist}_{TS} = \frac{\sin Q_{TS} \times D_{TW}}{\sin \gamma_{TW}} \tag{5.19}$$

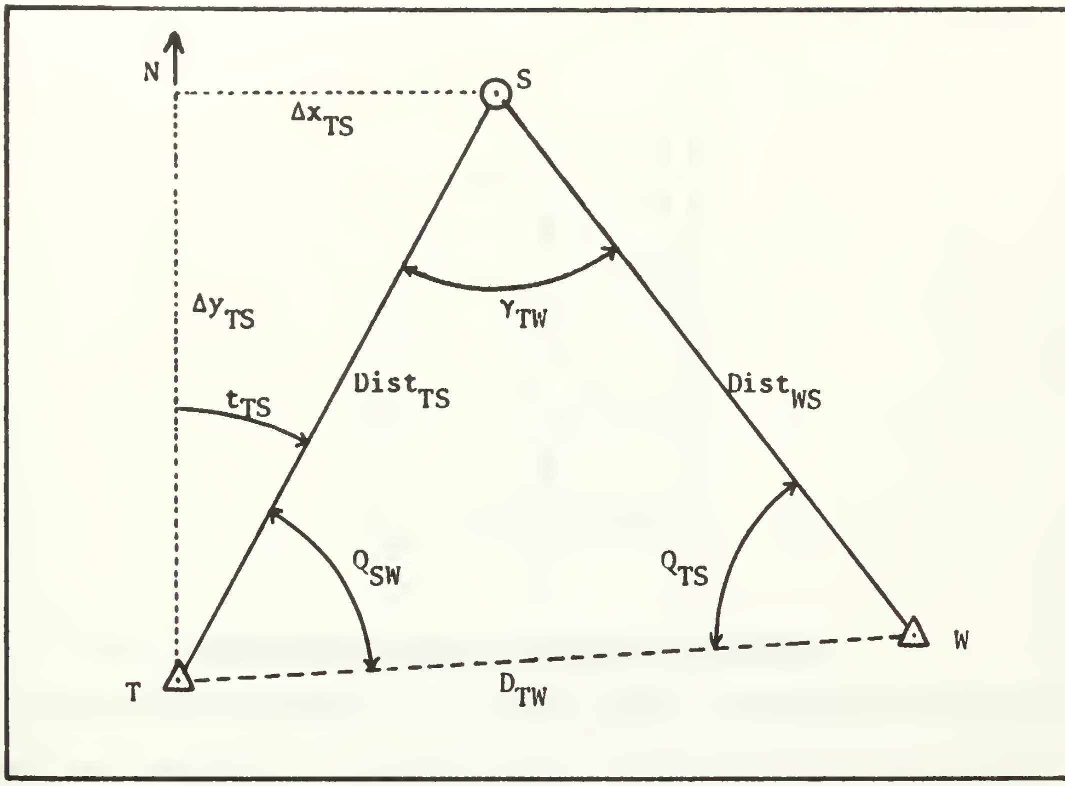

Figure 5.2. Geometric relationships for computation of approximate coordinates of ship position.

For the computation of approximate coordinates of the ship position S, the azimuth from terrestrial point T to S is denoted as t_{TS}. The increments Δx_{TS} and Δy_{TS} are then expressed, using (5.19), as:

$$\begin{aligned} \Delta x_{TS} &= Dist_{TS} \times \sin t_{TS} \\ \Delta y_{TS} &= Dist_{TS} \times \cos t_{TS} \end{aligned} \tag{5.20}$$

Thus the approximate coordinates of S are:

$$\begin{aligned} x_o &= x_T + \Delta x_{TS} \\ y_o &= y_T + \Delta y_{TS} \end{aligned} \tag{5.21}$$

5.4.2 Mathematical Structure.

The mathematical structure $L = F(X_a)$, the adjusted observations

as a function of the adjusted coordinates of the ship position, is now formed. It consists of the adjusted computed directions from each shore point to the ship position S.

$$
\begin{aligned}
t_{TS_a} &= \tan^{-1}\left[\frac{x_a - x_T}{y_a - y_T}\right] \\
t_{US_a} &= \tan^{-1}\left[\frac{x_a - x_U}{y_a - y_U}\right] \\
t_{VS_a} &= \tan^{-1}\left[\frac{x_a - x_V}{y_a - y_V}\right] \\
t_{WS_a} &= \tan^{-1}\left[\frac{x_a - x_W}{y_a - y_W}\right]
\end{aligned}
\qquad (5.22)
$$

5.4.3 Approximate Values of Observed Quantities.

By substituting the approximate unknown coordinates from (5.21) into the mathematical structure (5.22), the approximate numerical values of the observed quantities, L_o, are rigorously computed.

5.4.4 "Observed" Quantities.

Next the expressions for the observed quantities, L_b, are determined. It should be noted that L_b as referred to here is not simply directly observed directions, but a combination of the observed directions and orienting angles computed from the directions to known network stations. First the orienting angles O_i from each occupied shore station to two other known positions (see Figure 5.3) are calculated. These orienting angles are expressed as shown in (5.23) where the term β_{ij} for the computed azimuth from any occupied station i to any known network point j is:

$$\tan^{-1}\left[\frac{x_j - x_i}{y_j - y_i}\right]$$

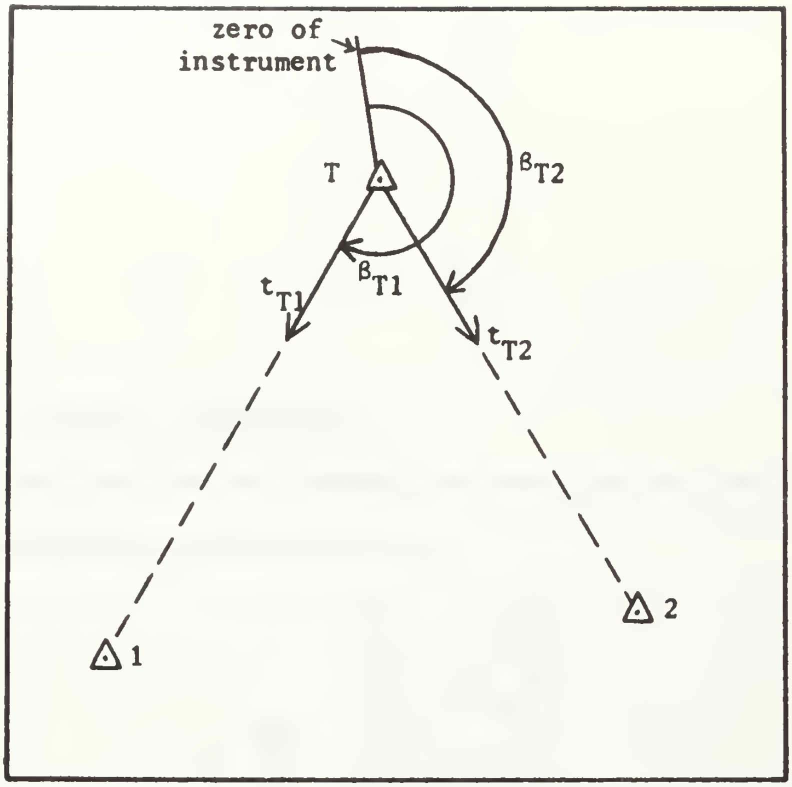

Figure 5.3. Geometric relationships for computation of orienting angles.

$$
\begin{aligned}
O_T &= \frac{(\beta_{T1} - t_{T1}) + (\beta_{T2} - t_{T2})}{2} \\
O_U &= \frac{(\beta_{U2} - t_{U2}) + (\beta_{U3} - t_{U3})}{2} \\
O_V &= \frac{(\beta_{V3} - t_{V3}) + (\beta_{V4} - t_{V4})}{2} \\
O_W &= \frac{(\beta_{W4} - t_{W4}) + (\beta_{W5} - t_{W5})}{2}
\end{aligned}
\tag{5.23}
$$

The desired "observed" values are then obtained by adding the observed ship directions to the orienting angles computed in (5.23). This vector, designated L_b, is shown on the following page.

$$L_b = \begin{bmatrix} t_{TS_b} + O_T \\ t_{US_b} + O_U \\ t_{VS_b} + O_V \\ t_{WS_b} + O_W \end{bmatrix} \tag{5.24}$$

5.4.5 Direction Coefficients.

The expressions for the elements of the direction coefficients array A are expressed symbolically as:

$$A = \frac{\partial L}{\partial X} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \tag{5.25}$$

where the individual elements, illustrated by a_{11} and a_{12}, take the following forms:

$$\begin{aligned} a_{11} &= \frac{\partial}{\partial x} \tan^{-1}\left[\frac{x_T - x_o}{y_T - y_o}\right] = -\frac{\rho''(y_T - y_o)}{(x_T - x_o)^2 + (y_T - y_o)^2} \\ a_{12} &= \frac{\partial}{\partial y} \tan^{-1}\left[\frac{x_T - x_o}{y_T - y_o}\right] = \frac{\rho''(x_T - x_o)}{(x_T - x_o)^2 + (y_T - y_o)^2} \end{aligned} \tag{5.26}$$

5.4.6 Weight Matrix.

Since the direction observations to the ship from each occupied shore position are independent of one another, the weight array P is a diagonal matrix. Assuming that all shore coordinates are free of error, for each observed direction to the ship the variance is

expressed as follows, as illustrated by the observation from station T:

$$m^2_{(TS + O_T)} = m^2_{TS} + m^2_{O_T} \tag{5.27}$$

The variance of the orientation, $m^2_{O_T}$, with n known network point orientation bearings, is given by:

$$m^2_{O_T} = \frac{1}{(n-1)^2} m^2_{T1} + \frac{1}{(n-1)^2} m^2_{T2} + \ . \ . \ . + \frac{1}{(n-1)^2} m^2_{T(n-1)} \tag{5.28}$$

In this problem, two known points are observed from each occupied station. Hence the orientation variance is:

$$m^2_{O_T} = m^2_{T1} \tag{5.29}$$

But the variance of an observation to the ship and to known network points is the same for the instrument at any particular station. Thus,

$$m^2_{TS} = m^2_{T1} = m^2_T \tag{5.30}$$

With (5.29) and (5.30), the observation variance given in (5.27) becomes:

$$m^2_{(TS + O_T)} = 2m^2_T \tag{5.31}$$

Therefore, for the four occupied stations, each with two orientation bearings, the weight matrix takes the following general form:

$$P = \frac{1}{2} \begin{bmatrix} \frac{1}{m_T^2} & 0 & 0 & 0 \\ 0 & \frac{1}{m_U^2} & 0 & 0 \\ 0 & 0 & \frac{1}{m_V^2} & 0 \\ 0 & 0 & 0 & \frac{1}{m_W^2} \end{bmatrix} \tag{5.32}$$

5.4.7 <u>Normals Matrix.</u>

The normal equations coefficients array N is formed by combining

array A from Article 5.4.5 and P from Article 5.4.6 according to the following formula as explained in the previous section.

$$N = A^T P A \tag{5.33}$$

5.4.8 Constant Vector.

The constant vector U, a column array, is formed as explained in Section 5.3 by the expression:

$$U = A^T P L \tag{5.34}$$

where A and P are developed in Articles 5.4.5 and 5.4.6. L, the alterations to the observed values of the observable quantities, is computed using (5.4). Its component vectors, L_o and L_b, are described in Articles 5.4.3 and 5.4.4.

5.4.9 Corrections to Unknown Parameters.

The alterations Δx and Δy to the approximations to the ship coordinates are computed by equations (5.18) wherein N and U are as described in the previous two articles.

5.4.10 Adjusted Values of Unknown Parameters.

The adjusted x and y coordinates of the ship position are calculated using (5.1) with quantities developed in Articles 5.4.1 and 5.4.9. In matrix form, this computation is expressed as follows:

$$X_a = \begin{bmatrix} x_a \\ y_a \end{bmatrix} = \begin{bmatrix} x_o + \Delta x \\ y_o + \Delta y \end{bmatrix} \tag{5.35}$$

5.4.11 Adjusted Values of Observations.

The adjusted directions L_a to the ship position are computed with

(5.5). The matrix array for this computation takes the following form:

$$L_a = \begin{bmatrix} {}^{1}a_T \\ {}^{1}a_U \\ {}^{1}a_V \\ {}^{1}a_W \end{bmatrix} = \begin{bmatrix} {}^{1}b_T + v_T \\ {}^{1}b_U + v_U \\ {}^{1}b_V + v_V \\ {}^{1}b_W + v_W \end{bmatrix} \tag{5.36}$$

5.5 ERROR ANALYSIS.

Using a procedure analagous to that described in Section 4.5, expressions for estimates of the variance-covariance matrices for the adjusted values of the unknown parameters and observed quantities are computed. These arrays indicate the accuracies of the adjusted direction observations and ship coordinates. The matrix notations employed in the previous section are continued in this development.

5.5.1 Variance of Unit Weight.

The variance of unit weight is expressed as:

$$m_o^2 = \frac{V^T P V}{\text{degrees of freedom}} \tag{5.37}$$

The weight matrix P is explained in Article 5.4.6. The vector of residuals of observations is computed using (5.9). The number of degrees of freedom is simply the number of direction observations less the number of unknowns, the x and y coordinates of the ship. For this problem, then:

$$\text{degrees of freedom} = 4 - 2 = 2 \tag{5.38}$$

5.5.2 Variance-Covariance Matrix of Unknown Parameters.

The variance-covariance array of the ship coordinates is given by:

$$\Sigma_X = m_o^2 Q_X \tag{5.39}$$

where Q_X, the weight coefficient matrix of the unknown parameters, is the inverse of the normals matrix N described in Article 5.4.7.

5.5.3 Variance-Covariance Matrix of Observations.

The weight coefficient matrix for the adjusted directions to the ship is expressed as:

$$Q_{L_a} = AN^{-1}A^T \tag{5.40}$$

where A and N are as described in Articles 5.4.5 and 5.4.7 respectively. Multiplying (5.40) by the variance of unit weight and simplifying the right side of the resultant expression by substituting (5.39), the estimate of the variance-covariance matrix of the adjusted observations becomes:

$$\Sigma_{L_a} = A\Sigma_X A^T \tag{5.41}$$

5.6 NUMERICAL EXAMPLE.

The solution for the determination of a ship position using four occupied shore stations is presented. Owing to the unavailability of actual survey data adaptable to this adjustment procedure, fictitious information has been generated for the problem.

Coordinates of all shore positions were arbitrarily chosen on a plotting sheet. Observed directions were computed to the network stations and the ship so as to permit the determination of realistic orienting angles.

5.6.1 Problem Input Information.

(a) Coordinates of occupied shore stations.

Data units: meters.

Station	X	Y
T	5000.0	15000.0
U	14000.0	17000.0
V	22500.0	20000.0
W	30000.0	23500.0

(b) Coordinates of other network stations used for orientation.

Data units: meters.

Station	X	Y
1	2500.0	4000.0
2	11000.0	6000.0
3	21500.0	9500.0
4	28000.0	13000.0
5	32500.0	16500.0

(c) Directions observed from occupied shore stations.

Source: generated using plotting sheet position coordinates.

Observing Station	To ship target	To orienting stations First	Second
T	34° 07' 50".5	195° 58' 20".2	149° 28' 40".4
U	15° 38' 40".3	205° 22' 20".1	145° 07' 00".3
V	346° 00' 05".2	200° 41' 30".7	157° 05' 35".9
W	321° 54' 40".7	210° 52' 05".3	180° 25' 50".2

(d) Standard error of theodolite observation: 0".5

Source: Manufacturer's handbook (Wild T2).

5.6.2 Auxiliary Information Computed by Program.

(a) Approximate ship position coordinates (in meters).

X	Y
15499.6	32500.1

(b) Orienting angles.

Station	Orienting angle
T	$-3^{\circ}\ 10'\ 04.''7$
U	$-10^{\circ}\ 07'\ 01.''0$
V	$-15^{\circ}\ 15'\ 03.''7$
W	$-20^{\circ}\ 05'\ 02.''9$

5.6.3 Adjusted Values and Associated Standard Errors.

(a) Position of ship (in meters).

X	m	Y	m
15499.84	0.12	32500.03	0.18

(b) Directions to ship position.

Observing Station	To ship target	m
T	$30^{\circ}\ 57'\ 48.''0$	$1.''6$
U	$5^{\circ}\ 31'\ 36.''9$	$1.''7$
V	$330^{\circ}\ 45'\ 02.''3$	$1.''6$
W	$301^{\circ}\ 49'\ 38.''0$	$1.''8$

6. SUMMARY AND CONCLUSIONS

6.1 SUMMARY.

Adjustment procedures have been applied to the two basic coastal hydrographic surveying problems; resection from the survey vessel and intersection from the shore. The adjustment procedures for each problem have been developed using the specific observational information and characteristics of the particular survey situation. In the first instance a generalized least squares adjustment computation method was employed and in the second the method of variation of parameters without constraints was used.

Background information concerning various aspects of coastal hydrography was presented as a basis for the types of observational procedures utilized. The various visual and electronic positioning systems and associated accuracy determination information were described to justify the particular data input and equation developments used in the adjustment problems.

Computer programs were written as the means for solving the adjustments. Information concerning the state of automation in coastal hydrography data acquisition and processing was presented to indicate how such problems could be solved by high-speed computers to provide the desired positional results to the hydrographer on a real-time basis. Modifications which would be necessary in order to utilize an existing automated survey system were outlined to indicate the feasibility of implementing these adjustment problems.

Certain other positioning systems; photogrammetric, satellite and laser; were described with their principal capabilities and limitations as applied to coastal hydrography. With the improvements being made in these and other positioning systems, it is envisioned that such methods may in the future replace or supplement the present day electronic and visual systems as the input data sources for problems such as are dealt with in this investigation.

Finally, numerical examples were worked using computer programs to indicate how the adjustment procedures could be applied to actual survey situations. Because real observational data was unavailable, information was generated for the purposes of these problems in as realistic a manner as possible. Production of this data included the use of plotting sheets and instruments utilized in practice in conventional coastal surveys. Arbitrary instrument and system accuracy estimates were used in the numerical examples, precluding their value in giving any more than a general indication of the accuracies which could be expected with such adjustment computation procedures.

6.2 RECOMMENDATIONS FOR FURTHER STUDY.

(a) Observational data from a single sounding line was utilized for the Chapter 4 adjustment computation. In practice, when sounding information is required, a series of such lines are usually run to obtain a satisfactory coverage of the area with depth determinations. Observations to shore points and electronic positioning data obtained during these additional runs along the same coastal region could be sequentially combined with the first group of data to strengthen the

results obtained for the shore positions.

(b) For the Chapter 4 adjustment procedure, it was assumed that no positional information was known concerning the prominent points along the shoreline used for sextant angles and compass bearings. Very likely, information would be available concerning at least a limited number of the terrestrial points, either as the result of earlier surveys or from aerial photography. The inclusion of such additional position information, provided it were at least as accurate as the data observed during the survey, would serve to improve the overall positional results determined through the adjustment.

(c) In the manner outlined in Chapter 4, the ship problem could be used to ascertain errors in electronic system lines of position caused by variations in electrical ground conductivity along coastal areas. For such determinations, all observations would be taken to geodetically determined points ashore. With the known shore positions, the need for compass directions, the weakest part of the visual observation procedure, would be eliminated. Such a procedure for determining fixed electronic system errors is given in (21). The method described therein utilizes only individual sextant angle positions, however, with no overall adjustment of data. This problem applied to their method would certainly appear to be a means of improving the positional information used to determine electronic system position line irregularities caused by ground conductivity variations.

(d) The use of photography of the coast taken from the ship would appear to be a means of improving solutions for the determination

of ship positions such as dealt with in Chapter 4. As mentioned in Chapter 2, terrestrial photogrammetry has been attempted on occasions with little success. However, with sufficient emphasis and attention devoted to developing a workable procedure, the use of photogrammetric information could enhance coastal survey positioning. Real-time film developing techniques and shipboard computers for data processing should allow photogrammetric methods to be integrated with the conventional procedures now employed in coastal survey operations.

6.3 CONCLUSIONS.

For coastal hydrographic survey situations wherein shore control is unavailable, the Chapter 4 adjustment procedure affords the hydrographer a means of obtaining both terrestrial and offshore positioning information. This adjustment method is primarily intended for use in a rapid, self-contained ship positioning system which could provide satisfactory positioning data along an inaccessible coast.

The actual position accuracies which would be obtained with this procedure must await evaluation of the method in a real survey situation. The positioning improvements may not be great enough to allow the results to compare with the accuracies requirements of conventional surveys conducted with the normal shore control. However, the accuracies will quite likely be better than the results which are now obtained using either visual or electronic methods separately in such situations.

When shore positions are known to geodetic accuracies, both adjustment procedures developed in this investigation can form the

bases for ship position determinations considerably better than are now obtained. While the present methods provide sufficient accuracies for nautical charting purposes, they are not satisfactory for certain precise offshore positioning requirements such as for engineering projects or emplacement of scientific instruments on the ocean floor.

Both adjustment procedures afford a means for the initial calibration and later monitoring of electronic positioning systems. With sufficient shore control, the offshore locations determined through these adjustments can also be used in analyzing irregularities in electronic position lines caused by ground conductivity variations as the radio waves propagate over adjacent terrestrial regions and then the coastal ocean surface.

As described earlier in this investigation, random and systematic errors are present in both the visual and electronic methods of positioning which render the Chapter 4 self-contained shipboard procedure unsuitable for more than approximate positioning sufficient to satisfy operational requirements. However, continued improvements are being made in electronic systems, particularly as concerns the determination of and correction for radio wave propagation irregularities which are now one of the greatest causes of positioning uncertainty. Also, other systems such as lasers and satellite navigation methods show promise of one day being capable of accuracies and characteristics which will render them useful in coastal survey operations. With improvements in existing methods and the anticipated new systems, there appears the strong possibility that these or similar adjustment procedures could in the future be utilized to provide the desired position information

with accuracies which meet the specifications required of the normal coastal hydrographic survey.

Adaptation of both adjustment procedures to survey situations wherein the coastal positions are accurately known will aid in providing offshore locations with considerably better accuracy than can now be accomplished with existing survey methods. While such increased accuracies are not required for the usual application of hydrography, the construction of nautical charts, they are required for certain other applications. With the greater attention being paid to all aspects of the use of the ocean and the coastal sea floor, numerous other precise positioning requirements not yet dreamed of may well arise which will necessitate the greater accuracies which these determinations can provide.

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